Formal Verification of Heap Space Bounds under Garbage Collection

Alexandre Moine
advised by Arthur Charguéraud and François Pottier

20/09/2024

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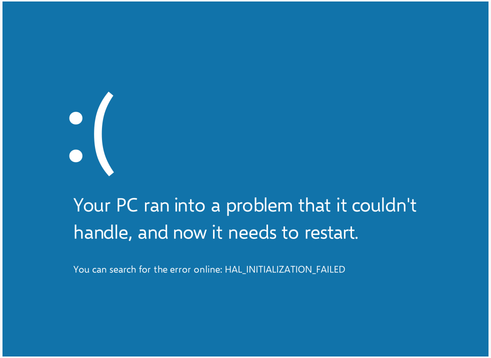

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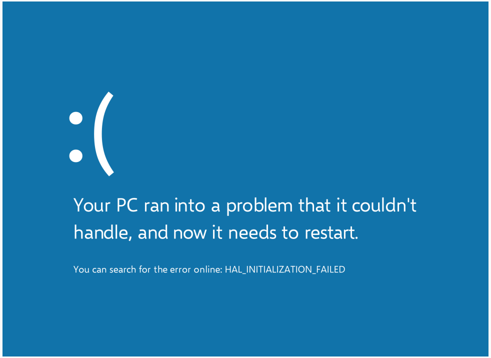


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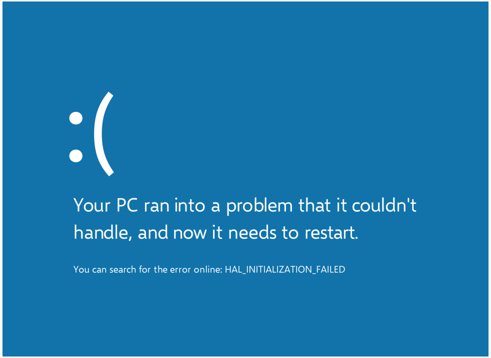

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How to ensure that a program has no bugs?

Focusing on a Bug Category

Correctness	$\rightsquigarrow$	The program does not compute the correct result.
Security	$\rightsquigarrow$	The program allows a thief to steal private data.
Resource usage	$\rightsquigarrow$	The program uses more resources than expected.

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Informal Central Question

How to bound the amount of memory required by a program?

A Reminder on Memory

- External memory for files
- RAM for runtime computations

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- the stack for data whose lifetime does not exceed the one of the allocating function
- the heap for everything else

- The stack stores data following a strict discipline.
  $\rightsquigarrow$ Establishing stack space bounds is well-studied [Carbonneaux et al., 2014].
- The heap is under the control of the programmer.
  $\rightsquigarrow$ Establishing heap space bounds a subtle task!

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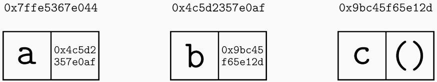

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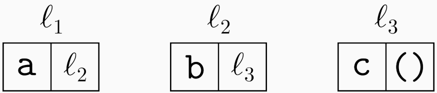

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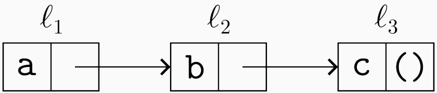

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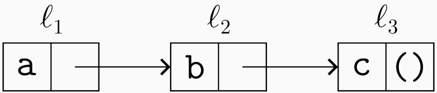

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- obtains the location of a fresh block,
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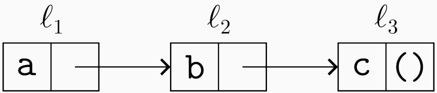

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Deallocation: with `free` / manual memory management vs. without `free` / garbage collection

Manual Memory Management

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char* arr = malloc(n);

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```
char* arr = malloc(n);
// -n bytes of available space
free(arr)
// +n bytes of available space
```

- ✔ Known techniques allow for proving heap space bounds: a resource meter can track the available heap space.
- ✖ Manual memory deallocation is error-prone: memory leak, use-after-free, double-free, etc.

Garbage Collection

Languages such as Java and OCaml have implicit deallocation.

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Central Question of my Thesis

How to establish heap space bounds in the presence of garbage collection?

Reachable Memory

```
let x = ref 42 in
let y = ref x in
x := 21;
let z = ref 84 in
y := z;
y
```

The heap

Reachable Memory

```
let x = ℓ_x in
let y = ref x in
x := 21;
let z = ref 84 in
y := z;
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The heap

ℓ_x

$\boxed{42}$

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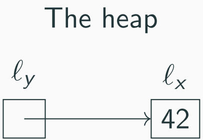

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ℓ_y := ℓ_z;
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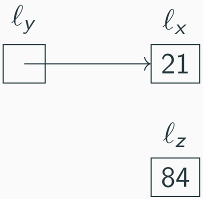

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ℓ_y

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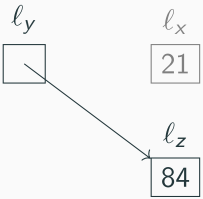

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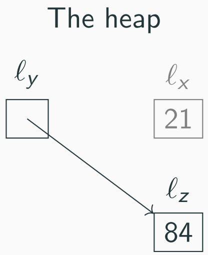

ℓ_y

Definitions

- The roots are the locations occurring in the program that remains to execute.
- The set of reachable blocks is computed from the roots following heap paths.

A More Elaborated Example

```
let rec mapsucc (xs : int list) : int list =
  match xs with
  | [] -> []
  | y::ys -> (y+1)::(mapsucc ys)
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What is the amount of free heap space needed by a call to `mapsucc xs`?

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- If `xs` is reachable from the calling context: $O(\text{length}\,\text{xs})$

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What is the amount of free heap space needed by a call to `mapsucc xs`?

The answer depends on the calling context!

- If `xs` is reachable from the calling context: $O(\text{length}\,\texttt{xs})$
- If `xs` is unreachable from the calling context: $O(1)$
  Cells from the input list can be freed as cells from the output list are allocated.

Goal: A Program Logic

We devise a variant of Separation Logic [O'Hearn et al., 2001, Reynolds, 2002].

$$\{\, \Phi \,\}\; t \;\{\, \lambda v.\; \Psi \,\}$$

Φ describes the heap before executing t. Ψ describes the heap after executing t.

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Standard reasoning rules:

$$\{\, \mathsf{True} \,\}\; \mathsf{alloc}\, 1 \;\{\, \lambda \ell.\, \ell \mapsto () \,\}$$

$$\{\, \ell \mapsto v \,\}\; \mathsf{load}\, \ell \;\{\, \lambda w.\, \ulcorner w = v \urcorner \;*\; \ell \mapsto v \,\} \qquad \{\, \ell \mapsto v \,\}\; \mathsf{store}\, \ell\, w \;\{\, \lambda ().\, \ell \mapsto w \,\}$$

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Can these rules be adapted to account for the available space under garbage collection?

Contribution

IrisFit, the first Separation Logic for verifying
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Including:

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- new realistic language constructs: protected sections and polling points, improving heap space bounds of lock-free data structures
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Part I: Heap Space Bounds for Sequential Programs

Based on

A High-Level Separation Logic for Heap Space under Garbage Collection

[Moine, Charguéraud, and Pottier; POPL'23]

Space Credits

Let $\Diamond 1$ represent one space credit [Hofmann, 1999].

- A space credit represents one free memory word.
- Space credits are splittable: $\Diamond(n_1 + n_2) \equiv \Diamond n_1 * \Diamond n_2$
- Space credits are not duplicable: $\Diamond n \not\Longrightarrow \Diamond n * \Diamond n$

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With manual memory management:

`alloc` consumes space credits

$$\{\, \Diamond 1 \,\}\ \mathsf{alloc}\, 1\ \{\, \lambda \ell.\ \ell \mapsto () \,\}$$

`free` produces space credits

$$\{\, \ell \mapsto v \,\}\ \mathsf{free}\, \ell\ \{\, \lambda().\ \Diamond 1 \,\}$$

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Madiot and Pottier [2022] use a low-level, assembly-like, language with an explicit stack.

Motivating Question

Can Madiot and Pottier's approach be scaled up to a high-level language?

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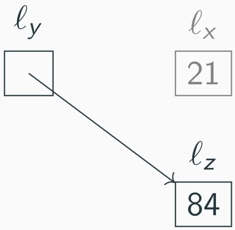

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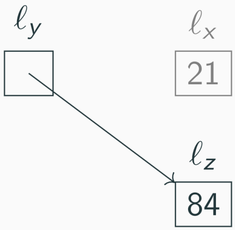

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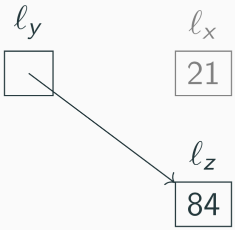

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1. ℓ is not a root $\rightsquigarrow$ the pointed-by-thread assertion
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The Pointed-by-Heap Assertion

The pointed-by-heap assertion [Kassios and Kritikos, 2013, Madiot and Pottier, 2022]

$$\ell \hookleftarrow A$$

- A is a multiset of locations.
- $\ell \hookleftarrow A$ asserts that A is an over-approximation of the predecessors of ℓ.
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We improve these assertions with fraction 0 and negative multisets. (Not shown here)

The Pointed-by-Thread Assertion

IrisFit features the pointed-by-thread assertion:

$$\ell \Lleftarrow \Pi$$

- Π is a set of thread identifiers. In a sequential setting, either $\{\pi\}$ or $\emptyset$.
- $\ell \Lleftarrow \{\pi\}$ asserts that ℓ may still be a root.
- $\ell \Lleftarrow \emptyset$ asserts that ℓ is not a root!

A Few Simplified Reasoning Rules

alloc produces points-to, pointed-by-heap, and pointed-by-thread assertions:

$$\{\, \Diamond 1 \,\}\ \mathsf{alloc}\ 1\ \{\, \lambda\ell.\ \ell \mapsto () \ * \ \ell \hookleftarrow \emptyset \ * \ \ell \Lleftarrow \{\pi\} \,\}$$

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store updates pointed-by-heap assertions:

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load updates a pointed-by-thread assertion:

$$\{\,\ell \mapsto \ell' \ * \ \ell' \Lleftarrow \emptyset\,\}\,\text{load}\ \ell\,\{\,\lambda v.\ \ulcorner v = \ell' \urcorner \ * \ \ell \mapsto \ell' \ * \ \ell' \Lleftarrow \{\pi\}\,\}$$

Trimming and Logical Deallocation

When ℓ is not a root anymore, we can trim its pointed-by-thread assertion.

$$
\begin{array}{rl}
 & \{\, \ell \Lleftarrow \emptyset \;\; * \;\; \Phi \,\}\; t \;\{\, \Psi \,\} \;\; \land \;\; \ell \notin \mathit{roots}(t) \\
\Longrightarrow & \{\, \ell \Lleftarrow \{\pi\} \;\; * \;\; \Phi \,\}\; t \;\{\, \Psi \,\}
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A Small Example

```
let x = ref 66 in
let y = ref x in
y := (!x / 2);
let z = ref 9 in
!z + !y
```

- Correctness: what is the result of this program?
- Heap space bound: how much memory does it need?

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```
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```

```
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```

$\{\, \lambda v.\, \ulcorner v = 42 \urcorner \;*\; \Diamond 2 \,\}$

A Small Example, Verified

$\{\Diamond 2\}$

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```

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Two Specifications for mapsucc xs

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Soundness of IrisFit

Soundness Theorem

If $\{\, \Diamond S \,\}\ t\ \{\, \lambda\_.\mathrm{True} \,\}$ holds, then
the execution of the program t with a heap of size at least S

- cannot reach a stuck configuration, and
- cannot run out of memory.

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```
Lemma wp_adequacy (S:ℕ) (t t':tm) (σ:store) :
  locs t = ∅ →
  rtc step (t,∅) (t',σ) →
  (∀ `{!interpGS Σ},
      ⊢ ◊S -∗ wp t (λ _ ⇒ ⊤)) →
  not_stuck t' σ ∧ (live_heap_size (locs t') σ ≤ S).
```

Part II: Scaling up to Concurrency

Based on

Will it Fit? Verifying Heap Space Bounds of Concurrent Programs under Garbage Collection with Separation Logic

[Moine, Charguéraud, and Pottier; submitted to TOPLAS]

Concurrency

- Modern computers are multi-core with shared memory.
- Threads execute concurrently and can be created dynamically.

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```
fork (fun () -> g x); f x
```

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f x   ||   g x
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f x   ||   g x
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Separation Logic scales seamlessly up to concurrency [O'Hearn, 2007, Jung et al., 2018].

Motivating Question

Can IrisFit be scaled up to concurrency?

How to Scale IrisFit up to Concurrency

Step 1: Annotate the triple with a ghost thread identifier π.

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Set of thread ids

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$$\{\,\ell \mapsto \ell' \ * \ \boxed{\ell' \Lleftarrow \{\pi_1\}}\,\}\ \pi_2 : \mathsf{load}\,\ell\ \{\,\lambda v.\ \ulcorner v = \ell' \urcorner \ * \ \ell \mapsto \ell' \ * \ \boxed{\ell' \Lleftarrow \{\pi_1, \pi_2\}}\,\}$$

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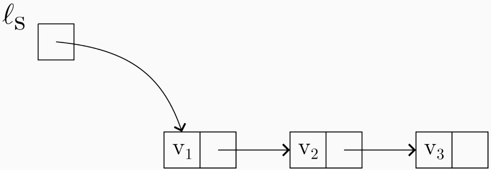
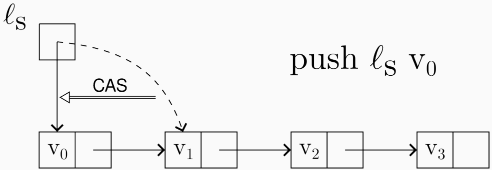
Step 3: Et voilà ?

- I verified some several concurrent programs: a lock, a concurrent counter, ...
- But certain lock-free data structures have an unexpected bound!

The Case of Lock-Free Data Structures: Treiber's Stack

A linearizable lock-free stack, implemented as a reference on an immutable list.





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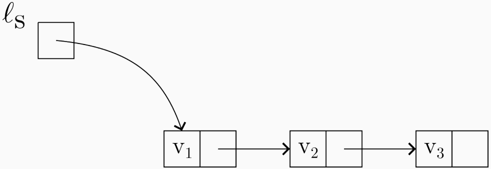

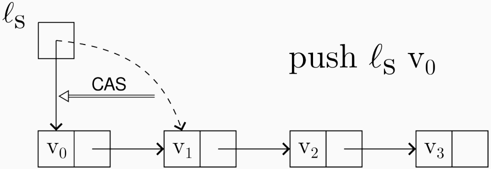


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let rec pop s =
  let xs = !s in
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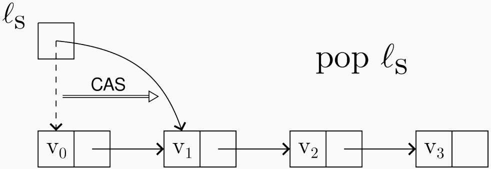

Desired Specification of Treiber's Stack (for Unboxed Values)

$$\left\langle \frac{\Diamond 2}{\forall L.\ \mathsf{stack}\ \ell_s\ L} \right\rangle \pi : \mathsf{push}\ \ell_s\ v \left\langle \frac{\lambda().\qquad \mathsf{True}}{\mathsf{stack}\ \ell_s\ (v :: L)} \right\rangle$$

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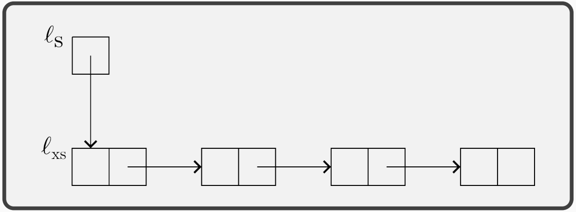
⚠ pop's specification is false: some interleavings invalidate it. ⚠

A Problematic Interleaving for pop

pop ℓ_s

||

pop ℓ_s;
pop ℓ_s;
pop ℓ_s;
a_big_alloc ()



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let xs = !ℓs in        ||  pop ℓs;
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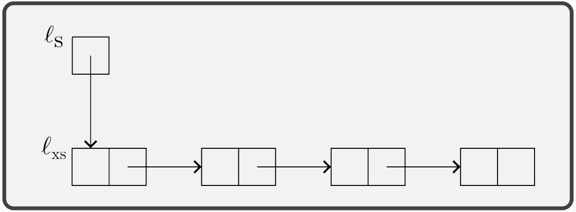

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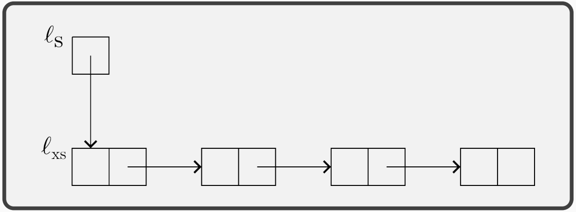

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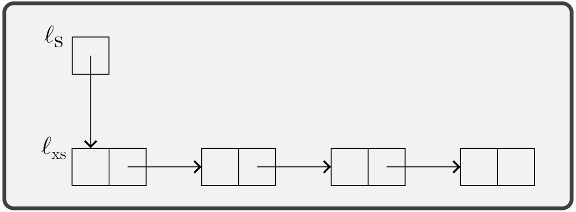

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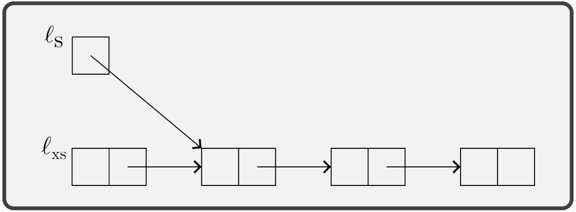

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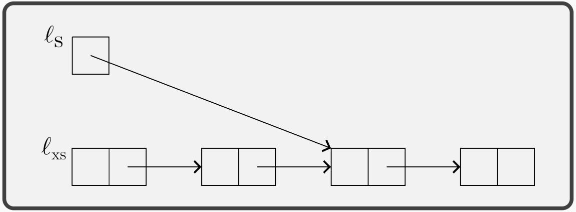

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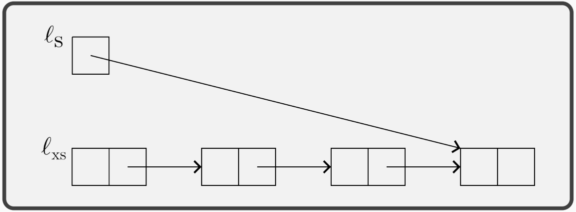

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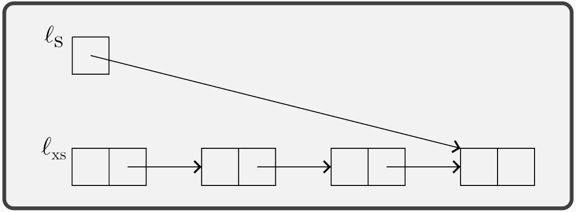

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Motivating Question

Can new language constructs be devised to prevent these interleavings?

Protected Sections

```
let xs = ℓ_xs in                                           ||  pop ℓ_s;
match xs with                                              ||  pop ℓ_s;
| nil -> assert false                                      ||  pop ℓ_s;
| y::ys -> if CAS s xs ys then y else pop s                ||  a_big_alloc ()
```

- The location ℓ_{xs} is a root for small number of instructions.
- The allocation should wait for these instructions to complete, until ℓ_{xs} is released.

Protected Sections

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let xs = ℓxs in
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```
let rec pop s =
  enter (); let xs = !s in
  match xs with
  | nil -> assert false
  | y::ys -> if CAS s xs ys then (exit (); y) else (exit (); pop s)
```

$\rightsquigarrow$ The location ℓ_{xs} is a temporary root: it is a root only inside a protected section.

Protected Sections, in Details

- Each thread is either outside or inside a protected section.
- The GC runs only when every thread is outside protected sections.

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The set T is for temporary roots: T must be empty when the protected section ends.

Thread π_0	Thread π_1
`enter ();`	`enter ();`
`let xs = !`ℓ_s` in`	`let xs = !`ℓ_s` in`
`...`	`...`
	`CAS s xs ys`
`exit ();`	
`...`	`...`

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...	...
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```
exit ();
```
...

Thread π_1

```
enter ();
```
$\{ \boxed{\ell_s \mapsto \ell_{xs} * \ell_{xs} \hookleftarrow \{+\ell_s\} * \ell_{xs} \Lleftarrow \emptyset} * \text{inside}\,\pi_1\,\emptyset \}$
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...

$\{ \boxed{\ell_s \mapsto \ell_{xs} * \ell_{xs} \hookleftarrow \{+\ell_s\} * \ell_{xs} \Lleftarrow \emptyset} * \text{inside}\,\pi_1\,\{\ell_{xs}\} \}$
```
CAS s xs ys
```
$\{ \boxed{\ell_s \mapsto \ldots} * \ell_{xs} \hookleftarrow \emptyset * \ell_{xs} \Lleftarrow \emptyset * \text{inside}\,\pi_1\,\{\ell_{xs}\} \}$

$\{ \Diamond 1 * \text{inside}\,\pi_1\,\{\ell_{xs}\} \}$

...

Polling for Liveness

Due to protected sections, a thread may wait forever for the GC.

```
while true do                      ||
  enter (); ...; exit ()           ||     a_big_alloc ()
done                               ||
```

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- New construct: polling points.
- A thread facing a polling point stops its execution until no thread requires the GC.

```
while true do
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  poll ()                        ||    a_big_alloc ()
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An automatic approach guaranteeing liveness: a polling point in every loop.

Back to Reality

- Protected sections and polling points are inspired by safe points.
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Proposal

Ask the programmer to be explicit about protected sections;
let the compiler insert polling points.

There is More

In the manuscript:

- reasoning about closures
- case studies
- soundness proof
- simplified reasoning when no deallocation is needed
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Making use of the presented ideas, I participated in other projects:

POPL'24 *DisLog: A Separation Logic for Disentanglement* [Moine, Westrick, and Balzer]
a logic for "disentanglement", a reachability property of parallel programs.

ICFP'24 *Snapshottable Stores* [Allain, Clément, Moine, and Scherer]
verifying a data structure with non-trivial reachability arguments.

Future Work

Connections with related works

- Integration with verified compilers
  CakeML [Gómez-Londoño et al., 2020]
- Safe Memory Reclamation (SMR)
  Space consumption [Jung et al., 2023]
- Foundation for type systems
  AARA [Hoffmann and Jost, 2022]

Practical applications

- Protected sections for OCaml
- Control over polling points position

Theoretical extensions

- More advanced case studies
  Harris's list [Harris, 2001]
- Weak pointers and ephemerons

How to Establish Heap Space Bounds in the Presence of Garbage Collection?

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- space credits to keep track of available heap space
- pointed-by-heap and pointed-by-thread assertions to prove unreachability
- protected sections to improve heap space bounds of lock-free data structures
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Reasoning rules, case studies and soundness are mechanized.

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Thank you for your attention!

Backup Slides

The Bind Problem and its Solution

Trimming is unsound with the standard $\textsc{Let}$:
what if a location $\ell \in (roots(t_2) \setminus roots(t_1))$?

$$\frac{\{\,\Phi\,\}\; t_1 \;\{\,\Psi'\,\} \qquad \forall v.\; \{\,\Psi'\; v\,\}\; [v/x]t_2 \;\{\,\Psi\,\}}{\{\,\Phi\,\}\; \mathrm{let}\; x = t_1 \;\mathrm{in}\; t_2 \;\{\,\Psi\,\}}$$

✖ One could leak $\ell \Lleftarrow \{\pi\}$ in Φ.

- Unveiling fractions: $\ell \Lleftarrow_{(p_1+p_2)} (\Pi_1 \cup \Pi_2) \;\equiv\; \ell \Lleftarrow_{p_1} \Pi_1 \;*\; \ell \Lleftarrow_{p_2} \Pi_2$
- Only logical deallocation requires full fraction 1.
- The $\textsc{Let}$ rule withhold a fraction of the pointed-by-thread assertion.

$$\frac{roots(t_2) = \{\ell\} \qquad \{\,\Phi\,\}\; t_1 \;\{\,\Psi'\,\} \qquad \forall v.\; \{\,\ell \Lleftarrow_p \{\pi\} \;*\; \Psi'\; v\,\}\; [v/x]t_2 \;\{\,\Psi\,\}}{\{\,\ell \Lleftarrow_p \{\pi\} \;*\; \Phi\,\}\; \mathrm{let}\; x = t_1 \;\mathrm{in}\; t_2 \;\{\,\Psi\,\}}$$

Cycles

We handle cycles following the approach of Madiot and Pottier [2022].

$$\text{True} \mathrel{-\!\!*} \emptyset \,\text{☁}^{0}\, P$$

$$\begin{array}{r} D \,\text{☁}^{n}\, P \\ \ell \mapsto \vec{v} \;*\; \ell \leftarrowtail A \;*\; \ell \Lleftarrow \emptyset \end{array} \mathrel{-\!\!*} (\{\ell\} \cup D) \,\text{☁}^{n+size(\vec{v})}\, P \quad \text{if } A \subseteq P$$

$$D \,\text{☁}^{n}\, D \Rrightarrow \Diamond n \;*\; \big(\mathop{\Huge *}_{\ell \in D} \dagger\ell\big) \qquad \text{if } D \cap roots(t) = \emptyset$$

Closures

Functions with an environment are usually compiled down to closures.

A closure is	a heap allocated block	pointing to the environment's values.
Closure allocation	consumes space credits	and updates pointed-by assertions.

We encode closures as derived constructions using closure conversion:

- closure creation and call are not in the syntax,
- but we provide macros implementing them,
- and provide reasoning rules about these macros!

The True Pointed-by-Heap Assertion

$$\ell \mapsfrom_q A$$

(signed) multiset

$\in [0, 1]$

- $\ell \mapsfrom_1 A$ asserts that A is an over-approximation of the reachable predecessors of ℓ.
- $\ell \mapsfrom_1 \emptyset$ asserts that ℓ is unreachable from the heap.

$$\ell \mapsfrom_1 \{+\ell_1; +\ell_2\} \quad \mathrel{-\!\!*} \quad \ell \mapsfrom_{\frac{1}{2}} \{+\ell_1\} * \ell \mapsfrom_{\frac{1}{2}} \{+\ell_2\}$$

$$\ell \mapsfrom_{\frac{1}{2}} \{+\ell_1\} * \ell \mapsfrom_0 \{-\ell_1\} \quad \mathrel{-\!\!*} \quad \ell \mapsfrom_{\frac{1}{2}} (\{+\ell_1\} \uplus \{-\ell_1\})$$

Main invariant: if $\ell \mapsfrom_0 A$ then A must contain only negative elements.

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