An Efficient Pre-Determinization Algorithm

Cyril Allauzen and Mehryar Mohri

AT&T Labs – Research
180 Park Avenue, Florham Park, NJ 07932-0971, USA
{allauzen,mohri}@research.att.com
http://www.research.att.com/~{allauzen,mohri}

Abstract. We present a general algorithm, *pre-determinization*, that makes an arbitrary weighted transducer over the tropical semiring or an arbitrary unambiguous weighted transducer over a cancellative commutative semiring determinizable by inserting in it transitions labeled with special symbols. After determinization, the special symbols can be removed or replaced with ϵ-transitions. The resulting transducer can be significantly more efficient to use. We report empirical results showing that our algorithm leads to a substantial speed-up in large-vocabulary speech recognition. Our pre-determinization algorithm makes use of an efficient algorithm for testing a *general twins property*, a sufficient condition for the determinizability of all weighted transducers over the tropical semiring and unambiguous weighted transducers over cancellative commutative semirings. It inserts new transitions just when needed to guarantee that the resulting transducer has the twins property and thus is determinizable. It also uses a single-source shortest-paths algorithm over the min-max semiring for carefully selecting the positions for insertion of new transitions to benefit from the subsequent application of determinization. These positions are proved to be *optimal* in a sense that we describe.

1 Introduction

Weighted transducers are used in many applications such as text, speech, or image processing for the representation of various information sources [9, 11]. They are combined to create large and complex systems such as an information extraction or a speech recognition system using a general composition algorithm for weighted transducers [13].

The efficiency of such systems is dramatically increased when *subsequential* or *deterministic* transducers are used, i.e. weighted transducers with a unique initial state and with no two transitions sharing the same input label at any state. A general determinization algorithm for weighted transducers was introduced by [11]. The algorithm can be viewed as a generalization of the classical subset construction used for unweighted finite automata, it outputs a deterministic transducer equivalent to the input weighted transducer. But, unlike unweighted automata, not all weighted transducers can be determinized using that algorithm

– this is clear since some weighted transducers do not even admit an equivalent subsequential one, they are not *subsequentiable*.

We present a general algorithm, *pre-determinization*, that makes an arbitrary weighted transducer over the tropical semiring or an arbitrary unambiguous weighted transducer over a cancellative commutative semiring determinizable by inserting in it transitions labeled with special symbols. After determinization, the special symbols can be removed or replaced with ϵ-transitions. The resulting transducer can be significantly more efficient to use. We report empirical results showing that our algorithm leads to a substantial speed-up in large-vocabulary speech recognition.

Our pre-determinization algorithm makes use of an efficient algorithm for testing a *general twins property* [2], which is a characterization of the determinizability of functional finite-state transducers and that of unambiguous weighted automata over the tropical semiring or any cancellative commutative semiring, and also a sufficient condition for the determinizability of all weighted transducers over the tropical semiring.

The algorithm for testing the twins property determines some transitions whose presence violates the twins property. Transitions with new symbols need not be inserted at those positions however. There is some degree of freedom in the choice of those positions and their choice is critical to ensure greater benefits from the application of determinization. Our algorithm inserts new transitions just when needed to guarantee that the resulting transducer has the twins property and thus is determinizable. It uses a single-source shortest-paths algorithm over the min-max semiring for carefully selecting the positions for insertion of new transitions to benefit from the subsequent application of determinization. These positions are proved to be *optimal* in a sense that we describe.

2 Preliminaries

A *semiring* $(\mathbb{K}, \oplus, \otimes, \overline{0}, \overline{1})$ is a ring that may lack negation [10]. It has two associative operations $\oplus$ and $\otimes$ with identity elements $\overline{0}$ and $\overline{1}$. $\oplus$ is commutative, $\otimes$ distributes over $\oplus$ and $\overline{0}$ is an annihilator for $\otimes$. A semiring is said to be *commutative* when its multiplicative operation $\otimes$ is commutative. A commutative semiring is said to be *cancellative* when for all a, b, c in $\mathbb{K}$ with $c \neq \overline{0}$, $a \otimes c = b \otimes c$ implies $a = b$. The *tropical semiring* $(\mathbb{R}_+ \cup \{\infty\}, \min, +, \infty, 0)$ or the *real semiring* $(\mathbb{R}, +, \times, 0, 1)$ are classical examples of cancellative commutative semirings.

A *weighted transducer* $T = (\Sigma, \Delta, Q, I, F, E, \lambda, \rho)$ over a semiring $\mathbb{K}$ is an 8-tuple where Σ is a finite input alphabet, Δ is a finite output alphabet, Q is a finite set of states, $I \subseteq Q$ the set of initial states, $F \subseteq Q$ the set of final states, $E \subseteq Q \times \Sigma \times \Delta \times \mathbb{K} \times Q$ a finite set of transitions, $\lambda : I \to \mathbb{K}$ the initial weight function mapping I to $\mathbb{K}$, and $\rho : F \to \mathbb{K}$ the final weight function mapping F to $\mathbb{K}$ [15, 10]. *Weighted automata* can be defined in a similar way by simply omitting the output labels.

The results presented is this paper hold similarly for weighted transducers over the tropical semiring and unambiguous weighted transducers over a cancellative commutative semiring, cases where our algorithm for testing the twins property can be used [2]. However, to simplify and shorten the presentation, in the following, all definitions, proofs, and examples will be given for weighted transducers over the tropical semiring.

Given a transition $e \in E$, we denote by $i[e]$ its input label, $o[e]$ its output label, $w[e]$ its weight, $p[e]$ its origin or previous state and $n[e]$ its destination state or next state. Given a state $q \in Q$, we denote by $E[q]$ the set of transitions leaving q. A *path* $\pi = e_1 \cdots e_k$ in A is an element of E^* with consecutive transitions: $n[e_{i-1}] = p[e_i]$, $i = 2, \ldots, k$. We extend n and p to paths by setting: $n[\pi] = n[e_k]$ and $p[\pi] = p[e_1]$. A cycle π is a path whose origin and destination states coincide: $n[\pi] = p[\pi]$. We denote by $P(q, q')$ the set of paths from q to q' and by $P(q, x, q')$ and $P(q, x, y, q')$ the set of paths from q to q' with input label $x \in \Sigma^*$ and output label y (transducer case). These definitions can be extended to subsets $R, R' \subseteq Q$, by: $P(R, x, R') = \cup_{q \in R,\, q' \in R'} P(q, x, q')$. The labeling functions i (and similarly o) and the weight function w can also be extended to paths by defining the label of a path as the concatenation of the labels of its constituent transitions, and the weight of a path as the sum of the weights of its constituent transitions: $i[\pi] = i[e_1] \cdots i[e_k]$, $w[\pi] = w[e_1] + \cdots + w[e_k]$. The weight associated by a transducer T to an input string $x \in \Sigma^*$ and output string $y \in \Delta^*$ is:

$$[\![T]\!](x, y) = \min_{\pi \in P(I, x, y, F)} (\lambda[p[\pi]] + w[\pi] + \rho[n[\pi]]) \tag{1}$$

A *successful path* in a weighted transducer T is a path from an initial state to a final state. A state q of T is *accessible* if it can be reached from I. It is *coaccessible* if a final state can be reached from q. A weighted transducer T is *trim* if it contains no transition with weight ∞ and if all its states are both accessible and coaccessible. T is *unambiguous* if for any string $x \in \Sigma^*$ it admits at most one successful path with input label x. The *inverse* T^{-1} of a weighted transducer T is obtained by swapping the input and output labels of every transition in T and its *negation* $-T$ by negating the cost of every transition in T.

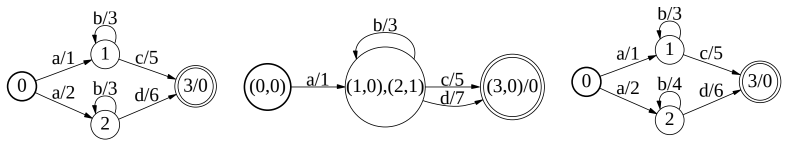
The result of the *composition* of two weighted transducers T_1 and T_2 over the tropical semiring is the weighted transducer defined as follows. States in the composition $T_1 \circ T_2$ of T_1 and T_2 are identified with pairs of a state of T_1 and a state of T_2.[1] Leaving aside transitions with ϵ inputs or outputs, the following rule specifies how to compute a transition of $T_1 \circ T_2$ from appropriate transitions of T_1 and T_2:[2]

$$(q_1, a, b, w_1, q_1') \;\text{ and }\; (q_2, b, c, w_2, q_2') \Longrightarrow ((q_1, q_2), a, c, w_1 + w_2, (q_1', q_2'))$$

When $T_2 = -T_1^{-1}$, we say that a state (q_1, q_2) of the composed transducer is a *diagonal state* if $q_1 = q_2$. Similarly, a transition is said to be a *diagonal transition*

[1] We use a *matrix notation* for the definition of composition as opposed to a *functional notation*.

[2] See [13] for a detailed presentation of the algorithm including the use of a filter for dealing with ϵ-paths.



(a) (b) (c)

Fig. 1. Determinization of weighted automata. (a) Weighted automaton over the tropical semiring A. (b) Equivalent weighted automaton B obtained by determinization of A.(c) Non-determinizable weighted automaton over the tropical semiring, states 1 and 2 are non-twin siblings.

when it is obtained by merging a transition (q_1, a, b, w_1, q_1') with its negative inverse $(q_1, b, a, -w_1, q_1')$ and more generally a path is said to be a *diagonal path* if all its constituent transitions are diagonal.

The following definitions will also be needed in the next sections [14]. An alphabet Σ can be extended by associating to each symbol $a \in \Sigma$ a new symbol denoted by a^{-1} and define Σ^{-1} as: $\Sigma^{-1} = \{a^{-1} : a \in \Sigma\}$. $X = (\Sigma \cup \Sigma^{-1})^*$ is then the set of strings written over the alphabet $(\Sigma \cup \Sigma^{-1})$. If we impose that $aa^{-1} = a^{-1}a = \epsilon$, then X forms a group called the *free group generated by Σ* and is denoted by $\Sigma^{(*)}$. Note that the inverse of a string $x = a_1 \cdots a_n$ is then simply $x^{-1} = a_n^{-1} \cdots a_1^{-1}$.

3 Determinization and the Twins Property

3.1 Determinization

A weighted automaton or transducer is said to be *deterministic* if it has a unique state and if no two transitions leaving the same state have the same input label. There exists a general determinization algorithm for weighted automata and transducers [11]. The algorithm is a generalization of the classical subset construction [1].

Figure 1 illustrates the determinization of a weighted automaton. The states of the output weighted automaton correspond to *weighted subsets* of the type $\{(q_0, w_0), \ldots, (q_n, w_n)\}$ where each $q_k \in Q$ is a state of the input machine, and w_k a remainder weight. The algorithm starts with the subset reduced to $\{(p, 0)\}$ where p is an initial state and proceeds by creating a transition labeled with $a \in \Sigma$ and weight w leaving $\{(q_0, w_0), \ldots, (q_n, w_n)\}$ if there exists at least one state q_k admitting an outgoing transition labeled with a, w being defined by: $w = \min\{w_k + w[e] : e \in E[q_k],\ i[e] = a\}$.

Similarly, Figure 2 illustrates the determinization of a finite-state transducer. Here, the states of the resulting transducer are *string subsets* of the type $\{(q_0, x_0), \ldots, (q_n, x_n)\}$, where each $q_k \in Q$ is a state of the input machine, and x_k a remainder string. We refer the reader to [11] for a more detailed presentation of these algorithms.

Unlike the unweighted automata case, not all weighted automata or finite-state transducers are *determinizable*, that is the determinization algorithm does

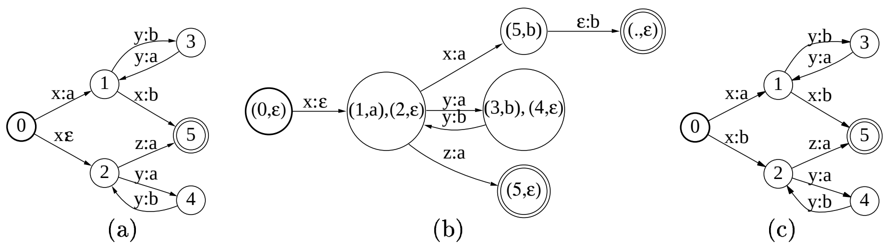


Fig. 2. Determinization of finite-state transducers. (a) Finite-State transducer T. (b) Equivalent transducer T' obtained by determinization of T. (c) Non-determinizable finite-state transducer, states 1 and 2 are non-twin siblings

not halt with some inputs. Figure 1(c) shows an example of a non-determinizable weighted automaton and Figure 2(c) a non-determinizable finite-state transducer. Note that the automaton of Figure 1(c) differs from that of Figure 1(a) only by the weight of the self-loop at state 2. The difference between that weight and that of the similar loop at state 1 is the cause of the non-determinizability.

3.2 The twins property

There exists a characterization of the determinizability of weighted transducers based on a *general twins property* and an efficient algorithm for testing that property under some general conditions [11, 2].

The twins property was originally introduced by [6, 7, 5] to give a characterization of the determinizability of unweighted functional finite-state transducers.[3] The definition of the twins property and the characterization results were later extended by [11] to the case of cycle-unambiguous weighted automata. The general twins property for weighted transducers presented here combines both sets of definitions and characterizations [2].

Two states q and q' are said to be *siblings* when they can be reached from the initial states I by paths sharing the same input label and when there exists a cycle at q and a cycle at q' labeled with the same input. Figure 3(a) illustrates this definition. Two sibling states q and q' of a weighted finite-state transducer are said to be *twins* if the two following conditions:

$$o[\pi]^{-1}o[\pi'] = o[\pi c]^{-1}o[\pi' c'] \tag{2}$$

$$w[c] = w[c'] \tag{3}$$

hold for any paths π from I to q and π' from I to q', and for any cycles c in q and c' in q' such that $i[\pi] = i[\pi']$ and $i[c] = i[c']$. T is said to have the *twins property* if any two siblings in T are twins. Note that in this definition q may be equal to q' and that we may have $\pi = \pi'$ or $c = c'$, or that π or π' can be the empty path if q, or q', is the initial state.

[3] The twins property was recently shown to provide a characterization of the determinizability of all unweighted finite-state transducers [3].

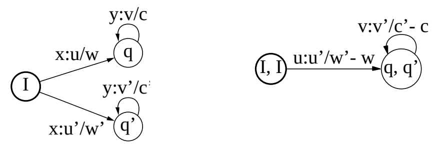


Fig. 3. (a) Two sibling states q and q' in T. (b) The corresponding configuration in $-T^{-1} \circ T$.

In the case of weighted automata, only condition 3 on the equality of the cycle weights is required, and in the case of unweighted transducers, only condition 2 on the output labels. The twins property is a sufficient condition for the determinizability of weighted automata or weighted transducers over the tropical semiring [11]. It is a necessary and sufficient condition for the determinizability of unweighted transducers [3] and that of unambiguous weighted automata or weighted transducers over the tropical semiring [11, 2].

Polynomial-time algorithms were given by [16, 4] to test the twins property for unweighted transducers. An efficient algorithm for testing the twins property for weighted and unweighted transducers was given by [2]. The algorithm is based on the composition of T with its negative inverse $-T^{-1}$. Assume that T is a trim cycle-unambiguous weighted transducer over the tropical semiring, then T has the twins property if and only if the following conditions hold for any state q, any path π from the initial state to q, and any cycle c at q in $-T^{-1} \circ T$ [2]:

$$i[\pi]^{-1}o[\pi] = i[\pi c]^{-1}o[\pi c] \tag{4}$$

$$w[c] = 0 \tag{5}$$

Figures 3(a)-(b) illustrate these conditions.

4 Pre-Determinization Algorithm

This section describes a general algorithm, *pre-determinization*, to make an arbitrary weighted transducer T over the tropical semiring or an arbitrary unambiguous weighted transducer T over a cancellative commutative semiring determinizable. The key steps of our algorithm are the following. We first augment the algorithm for testing the twins property for weighted transducers to mark the transitions of the transducer $-T^{-1} \circ T$ that are found by the algorithm to violate the twins property with distinct markers. These markers are then used to *disconnect* some paths of $-T^{-1} \circ T$ by inserting transitions with special symbols in T. We use a shortest-first algorithm over a min-max algorithm to disconnect simple cycles at the best position and in the desired order of visit of the simple cycles.

4.1 Marking transitions of the composed transducer

The algorithm for testing the twins property computes the composed transducer $S = -T^{-1} \circ T$ and determines paths violating condition (4) or (5). We augment this algorithm to mark the transitions of S found to violate these conditions with distinct markers. More precisely, we use the following markers. If a transition e in S is marked by

i) M_l, then there exist a cycle c containing e and a path π such that the label condition (4) does not hold;
ii) M_w, then there exist a cycle c containing e and a path π such that the weight condition (5) does not hold;
iii) M_a, then there exists a path π_0 containing e such that the label condition (4) does not hold for all cycles c accessible by a path π admitting π_0 as a prefix.

Markers are not exclusive, a transition may be marked with several markers or none. We denote by $M[e]$ the set of markers assigned to a transition e by the augmented test of the twins property.

4.2 Disconnecting Paths

By definition of composition, a path $\pi = e_1 \cdots e_n$ in the composed transducer S is the result of matching the input label of a path $\pi_1 = e_1^1 \cdots e_1^n$ of T with the input label of a path $\pi_2 = e_2^1 \cdots e_2^n$ of T. Assume that π is not a diagonal path, then π can be eliminated from the composed machine S by inserting a new transition with a special symbol in π_1 or π_2, at any position i, $1 \leq i \leq n$, such that e_i is not a diagonal transition ($e_1^i \neq e_2^i$), since this would prevent π_1 or π_2 to match. We then say that path π has been *disconnected* and will often use the transition e_1^i (or e_2^i) to refer to the position of insertion of that special transition in T. The choice of the position e_1^i (or e_2^i) is critical for the subsequent application of determinization and will be discussed in detail in Section 4.3.

Proposition 1 (Correctness). *Let T be a weighted transducer over the tropical semiring or an unambiguous weighted transducer over a cancellative commutative semiring, let S be the corresponding composed transducer, and let T' be the transducer obtained from T after application of the following operations:*

1. *if $M[e] \cap \{M_w, M_l\} \neq \emptyset$, disconnect all simple non-diagonal cycles containing e in S.*
2. *if $M[e] \cap \{M_l\} \neq \emptyset$, disconnect all simple non-diagonal paths from an initial state leading to a diagonal cycle containing e in S.*
3. *if $M[e] \cap \{M_a\} \neq \emptyset$, disconnect all simple non-diagonal cycles in S reachable from e, and all simple non-diagonal paths containing e in S from the initial state to a diagonal cycle.*

Then T' has the twins property and if we replace the special symbols in T' by ϵ, then T' becomes equivalent to T.

Proof. The proof follows directly the definition of the twins property and the proof of the correctness of the algorithm to test for the twins property from [2]. □

In what follows, we will focus on the algorithm for disconnecting all the simple non-diagonal cycles containing a transition e in S with $M[e] \cap \{M_w, M_l\} \neq \emptyset$ (the first item of Proposition 1). A similar algorithm can be used to disconnect the paths leading to a diagonal cycle containing a transition e with $M[e] \cap \{M_l\} \neq \emptyset$ (second item). Disconnecting the paths defined by the third item of Proposition 1 can be done using the same algorithms. It first requires determining all the strongly connected components reachable from a transition e with $M[e] \cap \{M_a\} \neq \emptyset$. This can be done in time linear in the size of S by computing a topological order of the component graph of S [8].

4.3 Positions for Insertion of Transitions

As mentioned earlier, different positions can be chosen to disconnect a non-diagonal simple cycle C of S. Our choice is motivated by the subsequent application of determinization, that is, we wish determinization to merge the longest possible paths to improve the efficiency of use the resulting transducer.

For any transition e in T, we define its *merging power*, $m[e]$, as the minimum length of the paths that can be merged with a path containing e if a special symbol is inserted at e. Thus, if the choice is between two transitions e_1 and e_2 for the insertion of a special symbol, with $m[e_1] < m[e_2]$, e_2 is preferable since it can allow longer paths to be merged. We then say that e_2 is a *more favorable position for determinization* than e_1.

Since composition merges pairs of paths with matching labels, the merging power of a transition e can be naturally defined in terms of the composed transducer S. Let E_S denote the set of transitions of S and denote by (e, e') a transition of E_S obtained by matching the negative inverse of the transition e and the transition e' in composition. The level of each transition $(e, e') \in E_S$ in a breadth-first search tree of S can be computed in linear time in the size of S [8]. Denote by $L[(e, e')]$ the level of (e, e'). For any transition e in T, let $\Phi[e]$ be the set of non-diagonal transitions of E_S obtained by matching e with some other transition e'. The merging power of a transition e of T can then be defined by:

$$m[e] = \begin{cases} \min\{L[(e', e'')] : (e', e'') \in \Phi[e]\} & \text{if } (\Phi[e] \neq \emptyset) \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

And a simple cycle C in S should be disconnected at a transition (e, e') such that e (or e') is the most favorable position for determinization:

$$e = \operatorname{argmax}\{m[e] : \Phi[e] \in C\} \tag{7}$$

Since disconnecting one cycle may affect another, it is also important to determine in what order simple cycles are disconnected. To avoid disconnecting a

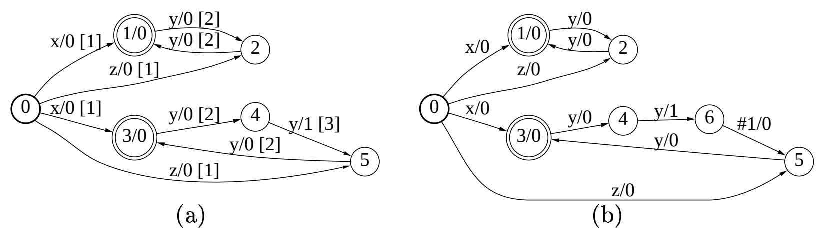


Fig. 4. (a) Non-determinizable weighted automaton A over the tropical semiring. The merging power $m[e]$ of each transition e is indicated in square brackets. (b) Weighted automaton B, output of the pre-determinization algorithm applied to A.

cycle more than once, we must start with the simple cycle whose most favorable insertion position e has the minimum merging power. Note that the max operation is used to determine the most favorable position along a cycle and the min operation to determine the order in which these cycles are visited and disconnected.

For each strongly connected component, we must disconnect either all cycles or each simple non-diagonal cycle containing a transition marked with M_w or M_l in the order just defined. Enumerating all simple cycles explicitly to disconnect them can be very costly. Instead, since the operations used are min and max and since the min-max semiring is 0-closed, we can use a single-source shortest-distance algorithm over $(\mathbb{N} \cup \{\infty\}, \min, \max, \infty, 0)$ to visit and disconnect simple cycles in the desired order [12]. For the purpose of determining the order of visit of the cycles, we can assign to each transition $(e_1, e_2) \in E_S$ the weight $\max\{m[e_1], m[e_2]\}$. By definition, the *shortest-first* order of this algorithm then coincides exactly with the desired order and guarantees that all simple cycles are visited as described. A simple cycle is disconnected at the transition with the maximum merging power if it was marked to be disconnected and is not already disconnected as a result of the disconnection of another cycle.

Proposition 2 (Optimality). *Let T be a weighted transducer over the tropical semiring or an unambiguous weighted transducer over a cancellative commutative semiring and let T' be the result of the application of the pre-determinization algorithm to T. Let e_s be a special transition in T' inserted at position e. e_s cannot be moved from e to a position e' in T' more favorable for determinization without violating the twins property.*

Proof. By definition of the pre-determinization algorithm, there exists a path π in S that contains a transition in $\Phi(e)$ and that must be disconnected for T' to have the twins property. If another position e' along π is selected for inserting e_s, then, by definition of the min-max single-source shortest paths algorithm, $m[e'] \leq m[e]$ and e' is not more favorable than e. If the position for the insertion of e_s is not along π then π is not disconnected and T' does not have the twins property. □

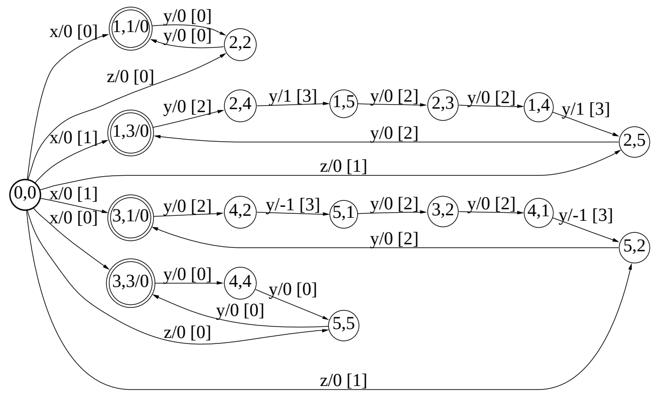


Fig. 5. The negative composition $-A^{-1} \circ A$ where A is the weighted automaton of Figure 4. For each transition (e_1, e_2), $\max\{m[e_1], m[e_2]\}$ is indicated in square brackets.

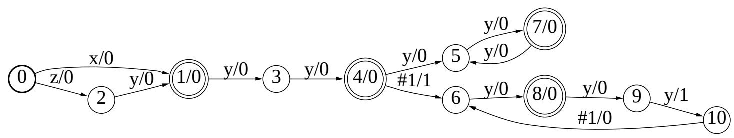


Fig. 6. The result of the determinization of the weighted automaton B of Figure 4(b).

Example. Let A be the weighted automaton over the tropical semiring shown in Figure 4(a). Figure 5 shows the composed automaton $-A^{-1} \circ A$. A does not have the twins property since $-A^{-1} \circ A$ admits non-zero cycles: the cycle at state $(1, 3)$ has weight 2 and the symmetric cycle at state $(3, 1)$ has weight -2. The algorithm for testing the twins property marks with M_l one of the transitions of each one of this cycles, e.g., the transitions from $(2, 5)$ and $(5, 2)$ labeled with y. A single-source shortest-distance algorithm over the min-max semiring from $(1, 3)$ determines the transition leaving state $(4, 1)$ as the position for the insertion of a special symbol since it has the largest value (3). This corresponds to inserting a new transition at the transition leaving state 4 in A. This insertion disconnects in fact both cycles with non-zero weight, thus no other disconnection is needed. Figure 4(b) shows B, the result of the application of the pre-determinization algorithm to A. B has the twins property and is thus determinizable. Figure 6 shows the automaton obtained by determinizing B.

4.4 Complexity

Let Q be the set of states and E the set of transitions of the weighted transducer T. In the worst case, the composed transducer $S = -T^{-1} \circ T$ may have as many as $|Q|^2$ states and $|E|^2$ transitions. The worst-case complexity of the algorithm for testing the twins property and marking the transitions is

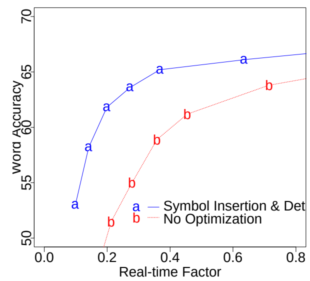


Fig. 7. Comparison of the optimization based on our symbol insertion algorithm and determinization versus no optimization in the 5,500-word vocabulary HMIHY 0300 task with a class-based language model.

quadratic in the size of S: $O(|Q|^2(|Q|^2 + |E|^2))$ [2]. The algorithm for disconnecting paths and cycles is based on a single-source shortest-distance algorithm over $(\mathbb{N} \cup \{\infty\}, \min, \max, \infty, 0)$ whose complexity is in $O(|Q|^2 \log |Q| + |E|^2)$ [12]. The algorithm also requires computing the component graph of S and its topological order which can be done in linear time in the size of S. Thus, the overall complexity of our pre-determinization algorithm is dominated by the test of the twins property and is $O(|Q|^2(|Q|^2 + |E|^2))$.

5 Experimental Results

We have fully implemented the algorithm described in the previous sections and measured its benefits by testing it in the 5,500-word vocabulary HMIHY 0300 speech recognition task. The class-based statistical language models used in that task are not determinizable and lead to other non-determinizable machines when combined with the weighted transducers representing information sources such as the pronunciation dictionary.

Our experiments showed that our algorithm leads to a substantial recognition speed-up in this task. Figure 7 gives recognition accuracy as a function of recognition time, in multiples of real-time on a single processor of a 1GHz Intel Pentium III Linux cluster with 256 KB of cache and 2 GB of memory. Using our algorithm, the accuracy achieved by the old non-optimized integrated transducer at .4 times real-time is reached by the new system using our optimization at about .15 times real-time, that is more than 2.6 times faster.

6 Conclusion

A general algorithm was presented that makes an arbitrary weighted transducer over the tropical semiring or any unambiguous weighted transducer over a cancellative commutative semiring determinizable by inserting in it auxiliary symbols and transitions just when needed to ensure that it has the twins property.

The auxiliary symbols are inserted at carefully selected positions to increase the benefits of the subsequent determinization. After determinization, the auxiliary symbols can be removed or simply replaced by the empty string.

Experiments in large-vocabulary speech recognition show that the resulting transducer can lead to a substantial recognition speed-up when the original weighted transducer is not determinizable.

References

1. A. V. Aho, R. Sethi, and J. D. Ullman. *Compilers, Principles, Techniques and Tools.* Addison Wesley: Reading, MA, 1986.
2. C. Allauzen and M. Mohri. Efficient Algorithms for Testing the Twins Property. *Journal of Automata, Languages and Combinatorics*, 8(2), 2003.
3. C. Allauzen and M. Mohri. Finitely Subsequential Transducers. *International Journal of Foundations of Computer Science*, to appear, 2003.
4. M.-P. Béal, O. Carton, C. Prieur, and J. Sakarovitch. Squaring transducers: An efficient procedure for deciding functionality and sequentiality. *Theoretical Computer Science*, 292:45–63, 2003.
5. J. Berstel. *Transductions and Context-Free Languages.* Teubner Studienbucher: Stuttgart, 1979.
6. C. Choffrut. Une caractérisation des fonctions séquentielles et des fonctions sous-séquentielles en tant que relations rationnelles. *Theoretical Computer Science*, 5:325–338, 1977.
7. C. Choffrut. *Contributions à l'étude de quelques familles remarquables de fonctions rationnelles.* PhD thesis, (thèse de doctorat d'Etat), Université Paris 7, LITP: Paris, France, 1978.
8. T. H. Cormen, C. E. Leiserson, and R. L. Rivest. *Introduction to Algorithms.* The MIT Press: Cambridge, MA, 1992.
9. K. Culik II and J. Kari. Digital Images and Formal Languages. In G. Rozenberg and A. Salomaa, editors, *Handbook of Formal Languages*, volume 3, pages 599–616. Springer, 1997.
10. W. Kuich and A. Salomaa. *Semirings, Automata, Languages.* Number 5 in EATCS Monographs on Theoretical Computer Science. Springer-Verlag, Berlin, Germany, 1986.
11. M. Mohri. Finite-State Transducers in Language and Speech Processing. *Computational Linguistics*, 23(2), 1997.
12. M. Mohri. Semiring Frameworks and Algorithms for Shortest-Distance Problems. *Journal of Automata, Languages and Combinatorics*, 7(3):321–350, 2002.
13. M. Mohri, F. C. N. Pereira, and M. Riley. Weighted Automata in Text and Speech Processing. In *Proceedings of the 12th biennial European Conference on Artificial Intelligence (ECAI-96), Workshop on Extended finite state models of language, Budapest, Hungary.* ECAI, 1996.
14. D. Perrin. Words. In M. Lothaire, editor, *Combinatorics on words*, Cambridge Mathematical Library. Cambridge University Press, 1997.
15. A. Salomaa and M. Soittola. *Automata-Theoretic Aspects of Formal Power Series.* Springer-Verlag: New York, 1978.
16. A. Weber and R. Klemm. Economy of Description for Single-Valued Transducers. *Information and Computation*, 118(2):327–340, 1995.