DIMACS, April 2006

UGLY PROOFS

and

BOOK PROOFS

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Tournament T on n players

Ranking σ

$\texttt{fit}$ = NonUpsets - Upsets

Erdős-Moon (1965): There exists T for all σ

$$\texttt{fit}(T, \sigma) \leq n^{3/2}\sqrt{\ln n}$$

Proof: *Random* Tournament

JS (1972, thesis!): For all T there exists σ

$$\texttt{fit}(T, \sigma) \geq cn^{2/3}$$

Proof: Random Sequential

Rank on Top or Bottom

JS (1980): For random T for all σ

$$\texttt{fit}(T,\sigma) \leq cn^{3/2}$$

Proof: Ugly

de la Vega (1983): Gem

Level 1: Top half against bottom half.

$\binom{n}{n/2}$ "different" σ; $n^2/4$ games

All $\texttt{1-fit} \leq c_1 n^{3/2}$

Level 2: $1-2$ or $3-4$ quartile games.

$< 4^n$ "different" σ; $n^2/8$ games

All $\texttt{2-fit} \leq c_2 n^{3/2}$

Level 3: $1-2$, $3-4$, $5-6$,$7-8$ octile games.

All $\texttt{3-fit} \leq c_3 n^{3/2}$

$\ldots \sum c_i$ *converges*

Six Standard Deviations Suffice

$A_1, \ldots, A_n \subseteq \{1, \ldots, n\}$

$\chi : \{1, \ldots, n\} \to \{-1, +1\}$, $\chi(A) := \sum_{a \in A} \chi(a)$

JS (1985): There exists χ

$$|\chi(A_i)| \leq 6\sqrt{n}, \mathbf{all} 1 \leq i \leq n$$

$b_i :=$ roundoff of $\chi(A_i)$ to nearest $20\sqrt{n}$

$\vec{b}(\chi) = (b_1, \ldots, b_n)$

(Boppana) b_i has low *entropy*

Subadditivity: $\vec{b}$ has low $(n\epsilon)$ entropy

$\Rightarrow$ Some $\vec{b}$ appears 1.99^n times

$\vec{b}(\chi_1) = \vec{b}(\chi_2)$ and differ in $\Omega(n)$ places

On the shoulders of Hungarians:

$$\text{Set } \chi = (\chi_1 - \chi_2)/2$$

$\Omega(n)$ colored, $|\chi(A_i)| \leq 10\sqrt{n}$

Iterate ...

ASYMPTOTIC PACKING

$k+1$-uniform hypergraph (e.g. $k=2$)

N vertices

$\deg(v) = D$

Any two v, w have $o(D)$ common hyperedges.

$N, D \to \infty$, k fixed

Conjecture (Erdős-Hanani) There exists a packing P with $|P| \sim N/(k+1)$

Rödl (1985): Yes!

JS (1995): Random Greedy Works

Continuous Time

Birthtime $b(e) \in [0, D]$

Packing P_t, Surviving S_t

$$\Pr[v \in S_t] \to f(t) = (1 + kt)^{-1/k}$$

History $H = H(v, t)$:

- $v \in e$, $b(e) \leq t \Rightarrow e \in H$
- $e \in H$, $e \cap f \neq \emptyset$, $b(f) < b(e) \Rightarrow f \in H$

History determines if $v \in S_t$

History is whp treelike and bounded

History $\sim$ Birth Process

Time backward t to 0

Start with root "Eve" (v)

Birth to k-tuplets Poisson intensity one

Children born fertile

Survival determined bottom up

Menendez Rule: If all k of birth survive, mother is killed

$f(t) := \Pr[\texttt{EveSurvives}]$

$f(t + dt) - f(t) \sim -f(t) \cdot dt \cdot f^k(t)$

$f'(t) = -f^{k+1}(t)$

$f(t) = (1 + kt)^{-1/k}$