Determining a function via the modulus of its derivative

Aris Daniilidis
(based on a joint work with D. Salas)

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Let $\mathcal{H}$ be a Hilbert space and $f, g : \mathcal{H} \to \mathbb{R}$ be two $\mathcal{C}^2$-smooth convex and bounded from below functions. In this setting one has:

$$\|\nabla f\| = \|\nabla g\| \qquad \text{if and only if} \qquad \nabla f = \nabla g.$$

In other words, every smooth convex and bounded from below function is determined (up to a constant) by the modulus of its derivative. The above result has been extended to nonsmooth (convex bounded from below) functions, using the remoteness of the subdifferential (*i.e.* its distance to zero), notion which coincides with the metric slope. In this talk we show that, surprisingly enough, the above results do not fundamentally rely neither on convexity nor on the linear structure of the ambient space. A consequence of our main result is that every Lipschitz, inf-compact function in a metric space is uniquely determined by its slope and its (metric) critical values.
If time allows, we shall discuss further extensions to more abstract settings.