**Social Networks**

**Lecture #3**

**Strong Triadic Closure.**

- In a social network, let \( f_1 \) and \( f_2 \) be two close friends of yours...
  - They are connected by strong ties to you.
  - Then, they (\( f_1 \) and \( f_2 \)) belong to a k-clan (for small k) with high prob.
  - Then, it is likely that \( f_1 \) and \( f_2 \) are acquaintances and are connected to each other by weak ties.

\[ \Pr\left[ (v, w) \in E \mid (u, v) \in E_s \land (u, w) \in E_s \right] > \Pr\left[ (v, w) \in E \right]. \]
TRIADIC CLOSURE PROPERTIES

- If \( f_1 \) and \( f_2 \) have a large subgroup of common friends (which include you),

  \( \Rightarrow \) Then it is probable that they are acquaintances.

  \( \Rightarrow \) The probability monotonically increases with the size of the set of mutual friends.

- In a connected social network, we expect to see lots of \( K_3 \)'s (Cliques of size 3).

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**Empirical Results.**

"Strength of Weak Ties" (1973)

Mark Granovetter: (American Sociologist, currently at Stanford Univ.)

- "Weak ties enable reaching populations and audiences with much higher efficiency than what is achievable or accessible via strong ties."

  **Why?**
Granovetter's PhD dissertation:
(Dept of Social Relation, Harvard Univ)

"Getting a Job"

Experiment:
Location: Newton, MA
Subjects: 282 professional, technical & managerial workers

Result: N = 54 = # individuals (out of 282) who found jobs through personal contacts.

- Strong Ties (16.6%)
- Weak Ties (83.4%)
  - Occasional Contacts (55.6%)
  - Rare Contacts (27.8%)
**Augmented Graph**

**Def**: Consider an "augmented" graph (undir.)

\[ G = (V, E, E_s) \]

in which

\[ E_s \subseteq E \subseteq V \times V. \]

\[ E = \text{The edges/ties.} \]
\[ \{ E \setminus E_s = \text{The weak ties.} \} \]
\[ E_s = \text{The strong ties.} \]

\[ \Rightarrow (u,v) \in E \Rightarrow u \text{ and } v \text{ are friends} \]
\[ \text{(acquaintances + close friends)} \]
\[ (u,v) \in E_s \Rightarrow u \text{ and } v \text{ are close friends} \]

The strong triadic closure property

if \((u,v) \in E_s\) and \((u,w) \in E_s\)

then \((v,w) \in E\) a.s.

\[ \Rightarrow Pr[(v,w) \in E \mid (u,v) \in E_s \land (u,w) \in E_s] \]
\[ > Pr[(v,w) \in E] \]

\[ \Rightarrow \text{Probability Raising in Social Network} \]
\[ \text{(Influence vs Social Influence)} \]
(I) \( \Pr \left[ (v, w) \in E \wedge (u, v) \in E_s \mid (u, w) \in E_s \right] \)

\( > \Pr \left[ (v, w) \in E \wedge (u, v) \in E_s \mid (u, w) \in E \setminus E_s \right] \)

(II) Consider a new relation \( R \)

\( \{ (u, v) \in R \} = \text{Event } u \text{ obtained a job through a "recommender" } v \).

\[ v \text{ recommended } u \text{ to } w; \]
\[ w \text{ verified } u \text{ independently; } \]
\[ w \text{ determined whether } u \text{ is a suitable candidate.} \]
\[
\Pr \left[ (u,v) \in R \mid (u,v) \in E_S \right] \\
\approx \Pr \left[ (u,w) \notin E \land (v,w) \in E_S \mid (u,v) \in E_S \right] \\
< \Pr \left[ (u,w) \notin E \land (v,w) \in E_S \mid (u,v) \in E \setminus E_S \right] \\
= \Pr \left[ (u,v) \in R \mid (u,v) \in E \setminus E_S \right] \\
\]

\( u = \text{Applicant} ; \ v = \text{Recommender} \)
\( w = \text{Verifier (Employer)} \)

**Strong Ties:**

\[
(u,v) \in E_S \land (v,w) \in E_S \\
\Rightarrow (u,w) \in E
\]

\( w \) knows about \( u \) and can use information in addition to what's provided in \( v \)'s recommendation.

**Weak Ties:**

\[
(u,v) \in E \setminus E_S \land (v,w) \in E_S \\
\Rightarrow (u,w) \notin E
\]

\( w \) will go by \( v \)'s recommendation only.