Q1. [5 ] Exercise #1 In $G(n, p)$ the probability of a vertex having degree $k$ is

\[ \binom{n}{k} p^k (1 - p)^{n-k}. \]

Show by direct calculation that the expected degree is $np$. Where is the mode of the binomial distribution? [Mode is the point at which the probability is maximum.] Compute directly the variance of the distribution.

Q2. [5 ] In $G(n, \frac{1}{n})$ what is the probability that there is a vertex of degree $\log n$? Give an exact formula; also derive simple approximations.

Q3. [10 ] What is the expected number of triangles and squares (3-cycles & 4-cycles) in $G(n, \frac{d}{n})$? What is the expected number of 4-cliques in $G(n, \frac{d}{n})$?