Q1. [10] Augment the signature \{¬, ∧\} by \lor and prove the completeness and soundness of the calculus obtained by supplementing the basic rules used so far with the rules:

\[
\begin{align*}
(\lor 1) & \quad \frac{X \vdash \alpha}{X \vdash \alpha \lor \beta, \beta \lor \alpha}; \\
(\lor 2) & \quad \frac{X, \alpha \vdash \gamma \mid X, \beta \vdash \gamma}{X, \alpha \lor \beta \vdash \gamma}
\end{align*}
\]

Q2. [10] Prove: (Finiteness Theorem for \models) If \( X \models \alpha \), then so too \( X_0 \models \alpha \) for some finite subset \( X_0 \subset X \).

Q3. [10] Using the preceding theorem, prove that if \( X \cup \{-\alpha \mid \alpha \in Y\} \) is inconsistent and \( Y \) is nonempty, then there exist formulas \( \alpha_0, \ldots, \alpha_n \in Y \) in \( Y \) such that

\[
X \vdash \alpha_0 \lor \cdots \lor \alpha_n.
\]