Once again, we visit several Islands in the Archipelago of Knights and Knaves along with our Antropologist. In these islands, those called knights always tell the truth and knaves always lie. Furthermore, each inhabitant is either a knight or a knave.

Notation: $K = \text{knight}, \neg K = \text{knave}$

Q1. [5] On one of these islands, an inhabitant makes the following two separate statements:

- There is gold on this island.
- If there is gold here, then there is diamond, too.

Is he a knight or knave? Does the island have gold? Does the Island have diamond?

Soln1. The statements are $S_1 \equiv G$ and $S_2 \equiv G \Rightarrow D \equiv \neg G \lor D$. Note that if $S_1$ is false then $S_2$ is true. So the only possibility is that $S_1$ and $S_2$ are both true. Thus the inhabitant is a knight and the island has gold as well as diamond.

Q2. [5] On one of these islands, the inhabitants will respond to any $k$-CNF for $k \geq 2$. Can you use this to find out if someone is a knight?

Soln2. Let the statement $S$ be $S \equiv K \lor \neg K$, a 2-CNF with one clause and also a tautology. Since only a knight will assert $S$ (which is always true), it is simple to determine if someone is a knight or knave.