Once again, we visit the Island of Knights and Knaves along with our Antropologist. In these islands, those called knights always tell the truth and knaves always lie. Furthermore, each inhabitant is either a knight or a knave.

Q1. [15] On another of these islands, there are seven comedians, who have agreed to do one-night standup gigs at two of the five hotels during a three-day festival, but each of them is available for only two of those days. The editor of Knight Times published the following schedule:

- Tomlin will do Aladdin and Caesars on days 1 and 2;
- Unwin will do Bellagio and Excalibur on days 1 and 2;
- Vegas will do Desert and Excalibur on days 2 and 3;
- Williams will do Aladdin and Desert in days 1 and 3;
- Xie will do Caesars and Excalibur on days 1 and 3;
- Yankovic will do Bellagio and Desert on days 2 and 3;
- Zany will do Bellagio and Caesars on days 1 and 2.

Note that there is a bit of an ambiguity about the exact schedule; for instance Tomlin may do Aladdin first and then Caesars (respectively on days 1 and 2), or in the opposite order – Caesars first and then Aladdin. However, it is believed that the editor of Knight Times is actually a knave; do you agree?

Soln1. Yes, the editor is a knave, because the schedule leads to a contradiction. First encode each schedule by a Boolean variable: t will mean that Tomlin will first do Aladdin [A1] and then Caesars [C2], while \( \neg t \) will mean the opposite order [C1] followed by [A1]. Thus:

\[ \neg(t \land w)[A1] \quad \neg(y \land z)[B2] \quad \neg(t \land z)[C2] \]
\[ \neg(w \land y)[D3] \quad \neg(u \land z)[B1] \quad \neg(\neg t \land x)[C1] \]
\[ \neg(v \land y)[D2] \quad \neg(u \land z)[E1] \quad \neg(\neg t \land y)[B2] \]
\[ \neg(\neg t \land x)[C1] \quad \neg(\neg v \land w)[D3] \quad \neg(\neg u \land \neg t)[E2] \]
\[ \neg(\neg u \land z)[B2] \quad \neg(\neg v \land \neg t)[C1] \quad \neg(\neg \neg t \land y)[D3] \]
\[ \neg(v \land x)[E3] \quad \neg(v \land x)[E3] \]

(1)
Each constraint is a Krom clause, giving rise to the following 2-SAT problem:

\[(\overline{t} \lor \overline{w}) \land (\overline{u} \lor \overline{\bar{g}}) \land (u \lor z) \land (\bar{g} \lor z) \land (t \lor x) \land (t \lor z) \land (\overline{x} \lor z) \land (\overline{t} \lor \overline{z}) \land (\bar{v} \lor y) \land (v \lor \bar{w}) \land (v \lor \bar{y}) \land (u \lor \bar{x}) \land (\bar{u} \lor v) \land (\bar{v} \lor \bar{x})\] (2)

There is a vicious cycle in the resulting Krom graph:

\[u \Rightarrow z \Rightarrow \bar{g} \Rightarrow \bar{t} \Rightarrow \bar{u} \Rightarrow z \Rightarrow \bar{t} \Rightarrow \bar{x} \Rightarrow u.\] (3)