In the Island of Knights and Knaves (IKK), those called \textit{knight}s always tell the truth and \textit{knaves} always lie. Furthermore, each inhabitant is either a knight or a knave.

In the class last time, we talked about the following incident:

An anthropologist, visiting this island, came across three inhabitants, whom he called, \textit{A}, \textit{B} and \textit{C}. He asked \textit{A}: “Are you a knight or a knave?” \textit{A} answered, but so indistinctly that the anthropologist could not understand what he said. He then asked \textit{B}: “What did he say?” \textit{B} replied: “He said that he is a knave.” At this point, \textit{C} piped up and said: “Don’t believe that; it is a lie!”

We concluded that \textit{B} is a knave and \textit{C} is a knight. Our reasoning was as follows:

Note that saying “I am a knave,” is same as saying “I am a liar,” which is self-referential and leads to a paradox (the so-called “Liar’s paradox.”) Thus \textit{B} is lying, when he says that \textit{A} said that he is a knave. Thus \textit{B} is a knave and \textit{C} is a knight. We can assume that \textit{A} must have said “I am a knight,” but that is not sufficient to determine if \textit{A} is a knight or a knave.

Q. 1. \ [+10 \ ] In a slight variant of this story, the anthropologist asked \textit{A} how many of the three are knaves.

Again \textit{A} answered indistinctly. So the anthropologist asked \textit{B} what \textit{A} had said. \textit{B} then said that \textit{A} had said that exactly two out of the three were knaves. As before, \textit{C} claimed that \textit{B} was lying.

Can you now tell if \textit{C} is a knight or a knave?

SOLN. 1 \ First note that \textit{B} and \textit{C} contradict each other:

\[ B \leftrightarrow \neg C. \]

Thus only one out of the two (\textit{B} or \textit{C}) is a knave. Thus if \textit{A} says “exactly two out of the three are knaves,” he is claiming to be a knave, which is impossible. Thus \textit{B} is a knave and \textit{C} is a knight. Since \textit{A} could not have also said that all three of them are knaves, he must have said “only one of us is a knave.” It is not possible to determine if \textit{A} is a knight or a knave.