A First-Order Theory of Communication and Multi-Agent Plans: Appendix A

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Appendix A: Changes to the theory and to the consistency proof

This document is appendix A to the paper, "A First-Order Theory of Communication and Multi-Agent Plans" by E. Davis and L. Morgenstern, to appear in *Journal of Logic and Computation* (henceforth, in this appendix, PLAN). In this appendix we first sketch the minor difference between the theory of action, knowledge, and communication given in PLAN and that given in the paper "Knowledge and Communication: A First-Order Theory" by E. Davis (2004b) (henceforth COMM). Second, we discuss how the proofs of the consistency theorems 1 and 2 of COMM can be modified and extended to give the proofs of theorems 1 and 2 of PLAN.

Logically,[1] PLAN is an extension of COMM, adding plans, multi-agent plans, and requests. However, PLAN also introduces some minor changes to the theory of time, actions, knowledge, and informative communication developed in COMM. The main change is the distinction drawn in PLAN between atomic "actions", such as "do(A,utter(C))" and high-level characterizations of actions, such as "inform(A, U, Q)". In COMM all these were considered "actions" and therefore we used notations such as "do(AS,inform(AH, U))." The motivation for introducing this distinction is that PLAN deals with knowledge preconditions. In Moore-style theories, this necessitates a distinction between rigid designators for actions (which are our "actions") and non-rigid designators (which are our "events"). The introduction of primitive actions in turns required a substantially stronger axiomatization of action theory. In particular, in PLAN we posit that an agent can execute only one action at a time, whereas in COMM, that axiom is false.[2]

Two other lesser changes should be noted. First, COMM assumes, for simplicity, that a communication was addressed to a single hearer, whereas PLAN allows a communication to be sent to a set of hearers, in order to accommodate the elevator problem, which was our starting point in PLAN. Second, PLAN introduces the ontological sort of additive durations — again, to accommodate the requirements of the elevator problem — which necessitates a larger set of temporal axioms.

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[1]Historically, it's actually the other way around; PLAN was written before COMM.

[2]Of course, if the theory of PLAN were to be extended to allow concurrent actions, that axiom would again become false, but it would be replaced by a more complex formulation, rather than just revert to the original unconstrained form in COMM.

We now turn to the consistency proofs; specifically how the proofs of theorems 1 and 2 of COMM are extended and modified to give proofs of theorems 1 and 2 of PLAN.

Construction of the model

In COMM definition 3 (communication successor): Replace "There exist agents ..." with "There exists an agent as, a set of agents u, and a situation sz such that s1≤sz and occurs(communicate(ah,u),s0,sz)."

Replace COMM definition 6 (acceptable physical theory) by PLAN definition 2.

COMM section A.2 (construction of the model of time, action, knowledge, and informative acts) remains almost unchanged. The only changes needed are:

- In COMM definition 10, replace the hearer agent AH by a set of hearer agents UA.
- In COMM definition 14, replace the third bulleted condition by the following:

  If there exists a tuple ⟨AS,UA,USSQ,SX⟩ in MM(USA) and OCCURS(COMMUNICATE(AS,UA),SX,PHYS(USA)) and agent A ∈ UA, then there exists a p-situation SXB and a tuple ⟨AS,UA,USSQ,SXB⟩ in MM(USB) and OCCURS(COMMUNICATE(AS,UA),SXB,PHYS(USB)).

These changes just reflect the change in the audience of a communicative act from a single hearer to a set of hearers.

Let us call the construction defined in COMM definition 16 an "informative model" (that is, a model of informative actions). The major change to the proof is added in between COMM section A.2 and COMM section A.3.; namely, the model of plans, multi-agent plans and requests is built on top of informative models. Our construction follows the structure of the comprehension axioms F.2, F.3, and F.4 of PLAN; that is, we will first define "simple plans" as a construction over informative models; then define "multi-agent plans" as a construction over simple plans; then broaden the informative model to include requests of multi-agent plans and commitments to simple plans; then define complex plans as constructions over the broadened models; then broaden the temporal model again to include commitments to complex plans.

Definition PLAN.17: Let TS be an informative model. A *closed interval* over TS is a pair ⟨S_START,S_END⟩ where S_START, S_END∈TS.U_SITS and S_START≤S_END. We will use the standard notation "[S_START, S_END]."

The extension of a simple plan P for agent AC has two parts: the set of all intervals over which P is begun and the set of all intervals over which P succeeds. These are subject to the constraint that AC controls only his own actions.

Definition PLAN.18: Let TS be an informative model and let AC be an agent. A *simple plan* P for AC over TS is a triple ⟨AC,BEGIN,SUCCEED⟩ where BEGIN and SUCCEED are sets of closed intervals over TS such that

- If [S0,S1]∈BEGIN and S0<SM<S1, then [S0,SM]∈BEGIN.
- SUCCEED is a subset of BEGIN.
- If [S0,S1]∈BEGIN and [S0,S1]∉SUCCEED, and S1 is not a choice point for AC, and S2 is a successor of S1 then [S0,S2]∈BEGIN.

BEGIN is the set of all intervals over which an execution of the plan begins, whether deliberate or not, and whether completed or abandoned. SUCCEED is the set of all intervals over which an execution of the plan succeeds, whether or not the agent executes the plan deliberately or knows at the end that it has succeeded.

Definition PLAN.19: Let TS be an informative model. A *multi-agent* plan MAP over TS is a mapping over AGENTS to simple plans such that, for each AC in AGENTS, MAP(AC) is a simple plan for AC over TS.

We now have to "broaden" our temporal model by adding acts of requesting multi-agent plans in the same way that we previously broadened the physical model by adding informative acts in COMM definitions 9-15. However, because the languages are layered — that is, multi-agent plans cannot contain request for multi-agent plans — the construction here is considerably simpler.

Definition PLAN.20: Let TS be an informative model. A *broadcast indicator* is a 6-tuple ⟨**BROADCAST**, AC,U,MAP,S1,S2⟩ where U is a set of agents, AC is an element of U, MAP is a multi-agent plan over TS, and S1, S2 are u-situations such that OCCURS(COMMUNICATE(AC,U),PHYS(S1),PHYS(S2)).

Definition PLAN.21: Let TS be an informative model. A *commit1 indicator* is a quintuple ⟨**COMMIT1**,AC,P,S1,S2⟩ where AC is an agent, P is a simple plan, and S1, S2 are u-situations such that S1<S2 and OCCURS(COMMUNICATE(AC,{AC}),PHYS(S1),PHYS(S2)).

Definition PLAN.22 A *request indicator* is either an broadcast indicator or a commit1 indicator.

Definition PLAN.23: Let TS be an informative model. Let US be a a u-situation in TS. An *r1-situation corresponding to US* is a pair ⟨US,SRI⟩ where SRI is a set of request indicators such that, for any tuple ⟨AC,U,MAP,US1,US2⟩ in SRI,

1. US1 precedes US and US2 is ordered with respect to US;
2. There is no different tuple in SRI with the same actor AC and the same starting situation S1.
3. There is no element of the form ⟨AS,U1,USSQ,S1⟩ in MM(US),

For any situation RS=⟨US,SRI⟩ we define USIT(RS)= US and PHYS(RS)=PHYS(US). (Conditions 2 and 3 enforce the conditions that an agent cannot simultaneously begin two request acts, or a request and an inform act.)

Definition PLAN.24: Let TS be an informative model. The corresponding *request-1 model* is the pair ⟨TS,RSS⟩ where RSS is the set of all r1-situations over TS.

Definition PLAN.25: Let RS be an request-1 model and let AC be an agent. A *complex plan* P for AC over RS is a triple ⟨AC,BEGIN,SUCCEED⟩ where BEGIN and SUCCEED are sets of closed intervals over RS such that

- If [S0,S1]∈BEGIN and S0<SM<S1, then [S0,SM]∈BEGIN.
- SUCCEED is a subset of BEGIN.
- If [S0,S1]∈BEGIN and [S0,S1]∉SUCCEED, and S1 is not a choice point for AC, and S2 is a successor of S1 then [S0,S2]∈BEGIN.

Definition PLAN.25 is identical to definition PLAN.18, except that it is taken over a request-1 model rather than an informative model.

Definition PLAN.26: Let TS be an informative model. A *commit-2 indicator* is a quintuple $\langle$**COMMIT2**,AC,P,S1,S2$\rangle$ where AC is an agent, P is a complex plan, and S1, S2 are r1-situations such that S1<S2 and OCCURS(COMMUNICATE(AC,{AC}),PHYS(S1),PHYS(S2)).

Definition PLAN.27: Let TS be an request-1 model. Let RS be a a r1-situation in TS. An *r2-situation corresponding to RS* is a pair $\langle$RS,CRI$\rangle$ where CRI is a set of commit-2 indicators such that, for any tuple $\langle$AC,U,MAP,RS1,RS2$\rangle$ in SRI,

1. US1 precedes US and US2 is ordered with respect to US;
2. There is no different tuple in SRI with the same actor AC and the same starting situation S1.
3. There is no element of the form $\langle$AS,U1,USSQ,S1$\rangle$ in MM(US),

If RS2 is an r2-situation corresponding to RS, we define PHYS(RS2)=PHYS(RS), and USIT(R2)=USIT(RS), RS=R1SIT(RS2).

Definition PLAN.28: Let RA=$\langle$USA,SRA$\rangle$ and RB=$\langle$USB,SRB$\rangle$ be two r-situations. RA *precedes* RB if:

- USA precedes USB; and
- SRA is the subset of request indicators in SRB whose starting time precedes USA.

Definition PLAN.29: Let RA, RB, be as above. Let AC be an agent. RB is *knowledge accessible from RA relative to AC* if the following conditions hold:

- USB is knowledge accessible from USA relative to AC.
- Let RI=$\langle$AS,U,MAP,US1,US2$\rangle$ be a request indicator in SRA. If US1 precedes USA and AC is in U, then RI is in SRB.
- Let RI=$\langle$AS,U,MAP,US1,US2$\rangle$ be a request indicator in SRB. If US1 precedes USB and AC is in U, then RI is in SRA.

Definition PLAN.30: A *reservation structure* is a quadruple consisting of

- A real value DELAY_TIME;
- A real value MIN_RESERVE_BLOCK;
- A relation RESERVED over AGENTS $\times$ AGENTS $\times$ CLOCKTIMES satisfying QD.1, Q.1, Q.2.
- A relation GOVERNS over AGENTS and ACTIONS satisfying axiom Q.3, Q.4.

Definition PLAN.31: A requestive model is a triple consisting of

- An informative model IS;
- The set of all r2-situations over IS;
- A reservation structure.

Definition of the interpretation

To construct the interpretation, we replace definition COMM:20 by the following:

Definition PLAN.32: (Long) Let $\mathcal{L}, \mathcal{M}, \mathcal{I}, \mathcal{W}, \mathcal{U}$ be as in COMM. We define the function $\mathcal{J}$ over the sorts and symbols of $\mathcal{W}$ as follows:

Sorts:

$\mathcal{J}$(the sort "clock time") = $\mathcal{J}$(the sort "duration") = the non-negative integers. (Strictly speaking, these should be tagged so that they are not actually the same individual.)

$\mathcal{J}$(the sort "agent") = $\mathcal{I}$("agent").

$\mathcal{J}$(the sort "set of agents") = power set of $\mathcal{I}$("agent")

$\mathcal{J}$(the sort "situation") = the set of r2-situations in $\mathcal{U}$.

$\mathcal{J}$(the sort "fluent") = the set of general fluents. See COMM Definition 17. Note that a "general fluent" involves a set of u-situations and *not* a set of r2-situations. This reflects the fact that comprehension axiom F.1 allows only formulas in $\mathcal{L}^1(\mathcal{D})$ to be used in the definition of fluents.

$\mathcal{J}$(the sort "physical fluent") = PF_IMAGES.

$\mathcal{J}$(the sort "u-interval") = The set of all unbounded intervals of situations; i.e. for any starting situation S0, a maximal totally ordered subset of the set of situations $S \geq S0$.

$\mathcal{J}$(the sort "physical actional") = $\mathcal{I}$("physical actional")

$\mathcal{J}$(the sort "physical action") = $\mathcal{I}$("physical action")

(Note: Unlike COMM, the theory here does not have an action "do(AS,inform(AH, Q))". Here "informative actions" will be the denotations of the corresponding utterances.)

$\mathcal{J}$(the sort "simple plan") = the set of simple plans.

$\mathcal{J}$(the sort "multi-agent plan") = the set of multi-agent plans

$\mathcal{J}$(the sort "complex plan") = the set of complex plans.

$\mathcal{J}$(the sort "utterance content") = **fluent** $\cup$ **simple plan** $\cup$ **multi-agent plan** $\cup$ **complex plan**

Let **informative actionals** be the set of all triples $\langle$**INFORM**,U,C$\rangle$ where U$\in$**set of agents** and C$\in$**fluent**

Let **commit1 actionals** be the set of all triples $\langle$**COMMIT1**,U,C$\rangle$ where U$\in$**set of agents** and C$\in$**simple plan**

Let **broadcast actional** $\equiv$ the set of all triples $\langle$**BROADCAST**, U, C$\rangle$ where U$\in$**set of agents** and C$\in$**multi-agent plan**.

Let **commit2 actionals** be the set of all triples $\langle$**COMMIT2**,U,C$\rangle$ where U$\in$**set of agents** and C$\in$**complex plan**

Let **utterance actionals** $\equiv$ **informative_actionals** $\cup$ **broadcast_actionals** $\cup$ **commit1_actionals** $\cup$ **commit2_actionals**.

Let **utterance actions** $\equiv$ $\{\langle$**DO**,ZU$\rangle$ | ZU$\in$**utterance actional**$\}$

$\mathcal{J}$(the sort "actional") = $\mathcal{I}$("physical actional") $\cup$ **utterance actionals**

$\mathcal{J}$(the sort "action") = $\mathcal{I}$("physical action") $\cup$ **utterance actions**

If σ is any other sort used in $\mathcal{L}$, then $\mathcal{J}(\sigma) = \mathcal{I}(\sigma)$.

Non-logical symbols:

The predicate $<$ and the function $+$ over clock times and duration have their regular denotation over the integers.

$\mathcal{J}$("<") (as a predicate on situations) = { $\langle S1, S2\rangle$ | $S1, S2 \in$ **situation** and $S1$ precedes $S2$. }

$\mathcal{J}$("holds") = { $\langle S, Q\rangle$ | $S \in$ **situation**, $Q = \langle PF, USS\rangle \in$ **fluent** and USIT(S) $\in USS$. }

$\mathcal{J}$("time") = { $\langle S, T\rangle$ | $S \in$ **situation**, $T \in$ **clocktime** and USIT(S) is of time T }.

$\mathcal{J}$("$\in$") = $\in$, whether as a predicate on agents and sets of agents or as a predicate on situations and u-intervals.

$\mathcal{J}$("communicate") = $\mathcal{I}$("communicate")

$\mathcal{J}$("utter") = {⟨U,C,Z⟩ | U∈**set of agents** ∧ C∈**utterance content** ∧ Z=⟨**UTTER**,U,C⟩.

$\mathcal{J}$("do") = $\mathcal{I}$("do") ∪ { ⟨ A,Z, ⟨**DO**,A,Z⟩⟩ | A ∈ **agent** and Z ∈ **utterance_actionals** }

$\mathcal{J}$("inform") = { ⟨A,U,Q, ⟨**DO**, A, ⟨**INFORM**,U,Q⟩⟩⟩ | A∈**agent**, U∈**set of agents** and Q∈**fluent** }

$\mathcal{J}$("commit1") = { ⟨A,P, ⟨**DO**, A, ⟨**COMMIT1**,P⟩⟩⟩ | A∈**agent** and P∈**simple plan** }

$\mathcal{J}$("broadcast_req") = { ⟨A,U,R, ⟨**DO**, A, ⟨**BROADCAST**,U,R⟩⟩⟩ | A∈**agent**, U∈**set of agents** and Q∈**multi-agent plan** }

$\mathcal{J}$("commit2") = { ⟨A,P, ⟨**DO**, A, ⟨**COMMIT2**,P⟩⟩⟩ | A∈**agent** and P∈**complex plan** }

$\mathcal{J}$("k_acc") = { ⟨A,S1,S2⟩ | A ∈**agents** and ⟨S1,S2⟩ ∈ K_ACC$_\infty$(A). }

$\mathcal{J}$("ck_acc") =
{ ⟨U,SA,SB⟩ |
  exists($S_0 = SA, S_1 \dots S_k = SB; A_1 \dots A_k \in U$) such that
    for ($i = 1 \dots k$) **k_acc**(A_i, S_{i-1}, S_i).
}.

Let **occurs_physical_action** = { ⟨E,RS1,RS2⟩ | E ∈ $\mathcal{I}$("physical action") and RS1,RS2∈**situation** and RS1<RS2 and OCCURS(E,PHYS(RS1),PHYS(RS2))].

Let **occurs_informative_action** =
{ ⟨E,RS1,RS2⟩ | RS1,RS2∈**situation** and $RS1 < RS2$ and
there exist (AS ∈ **agent**; U∈**set of agents**; Q1,Q2 ∈ **fluent**; USS1,USS2) such that
E=⟨**DO**,AS,⟨**INFORM**,U,Q1⟩⟩;
Q1=⟨PF1,USS1⟩, Q2=⟨PF2,USS2⟩;
USS2 = { US ∈ USS1 | ⟨U,US1,US⟩ ∈ **ck_acc** };
OCCURS(DO(AS,COMMUNICATE(U)),PHYS(RS1),PHYS(RS2)); and
⟨AS,U,USS2,PHYS(RS1)⟩ ∈ MM(USIT(RS2))
}

Let **occurs_commit1** =
{ ⟨E,RS1,RS2⟩ | RS1,RS2∈**situation** and $RS1 < RS2$ and E=⟨**DO**,A,⟨COMMIT1,P⟩⟩ and R1SIT(RS2) = ⟨US,SRI⟩ and ⟨**COMMIT1**,A,P,USIT(RS1),USIT(RS2)⟩ ∈SRI }

Let **occurs_broadcast** =
{ ⟨E,RS1,RS2⟩ | RS1,RS2∈**situation** and $RS1 < RS2$ and E=⟨**DO**,A,⟨BROADCAST,U,R⟩⟩ and R1SIT(RS2) = ⟨US,SRI⟩ and ⟨**BROADCAST**,A,U,R,USIT(RS1),USIT(RS2)⟩ ∈SRI }

Let **occurs_commit2** =
{ ⟨E,RS1,RS2⟩ | RS1,RS2∈**situation** and $RS1 < RS2$ and E=⟨**DO**,A,⟨COMMIT2,P⟩⟩ and
RS2 = ⟨R1SIT(RS2),SRI⟩ and ⟨**COMMIT2**,A,P,R1SIT(RS1),R1SIT(RS2)⟩ ∈SRI }

$\mathcal{J}$(“occurs”) = occurs_physical_action ∪ occurs_informative_action ∪ occurs_commit1 ∪ occurs_broadcast∪ occurs_commit2.

Let α be any symbol in $\mathcal{L}$ other than those enumerated above. $\mathcal{I}(\alpha)$ is a set of tuples of entities in $\mathcal{M}$. A tuple T' is a replacement for tuple T if, for each index I, U2P_MAP($T'[I]$) = $T[I]$. Then $\mathcal{J}(\alpha)$ is the set of all replacements R for the tuples in $\mathcal{I}(\alpha)$, such that any two situations in R are ordered under $\mathcal{J}$(“ < ”).

$\mathcal{J}$(“request”) = **broadcast_req** ∪ **commit1** ∪ **commit2**.

$\mathcal{J}$(”assignment”) = { ⟨R,A,P⟩ | R∈**multi-agent plan** and A∈**agent** and P∈**simple plan** and P=R(A) }.

$\mathcal{J}$(“plan”) = **simple plan** ∪ **complex plan**.

$\mathcal{J}$(“actor”) = { ⟨P,A⟩ | P∈**plan** and P=⟨A,BEGIN,SUCCEED⟩. }

$\mathcal{J}$(“next_step”) =
{ ⟨E,P,S1,S2⟩ |
P ∈ **plan** and S1,S2 ∈ **situation** and E∈**action** and ⟨S1,S2⟩ ∈ BEGIN and
there exist Z,SM,S3 such that E=⟨**DO**,**actor**(P),Z⟩ and S2<SM≤S3 and **occurs**(E,S2,S3) and ⟨S1,SM⟩∈BEGIN }

$\mathcal{J}$(“succeeds”) = {⟨P,S1,S2⟩ | P=⟨BEGINS, SUCCEED⟩ and ⟨S1,S2⟩ ∈ SUCCEED.

$\mathcal{J}$(“reserved”) = RESERVED

$\mathcal{J}$(“governs”) = GOVERNS.

$\mathcal{J}$(“delay_time”) = DELAY_TIME.

$\mathcal{J}$(“min_reserve_block”) = MIN_RESERVE_BLOCK.

The remaining primitives are all defined in terms of the above by definitional axioms. (Axioms Q.5 and Q.6 amount to a mutually recursive definition of “accepts_req” and “working_on”).

Proof of validity: Sketch

The proof of the validity of the theory relative to the model and the interpretation is a simple extension of the corresponding proof in COMM. The hard part of the proof in COMM was to show that the axioms of the physical theory were all valid in the extended model. This part of the proof is exactly the same here, since the extension here from p-situations to r-situation has the same structural properties as the extension in COMM from p-situations to u-situations. The proofs of the explicit axioms of time, action, knowledge, and informative actions that are the same, or nearly the same, in PLAN as in COMM carry over with minimal change. The axioms of action that are new here come within the category of physical axioms relative to the theory in COMM, and thus, if they hold in the physical theory, they hold in the extended theory. The validity of the axioms of comprehension and of speech acts (U.1—U.3, S.1—S.7, C.1—C.6) is just definition hunting. The theory of planning is mostly definitional (QD.1—QD.12). Of the proper axioms of the theory of planning: Q.1—Q.4 are simple properties of “reserved” and “govern”, built in to Definition PLAN.30; Q.5 and Q.6 are the mutually recursive definition of “working_on” and “accepts_req”; and Q.7 can be established using a simple recursive proof (in fact, we conjecture that Q.7 is provable from the other axioms.)