Davis-Putnam procedure: Example

Initial set of clauses S0:

1. P ∨ Q ∨ R
2. P ∨ ¬Q ∨ ¬R
3. P ∨ ¬W
4. ¬Q ∨ ¬R ∨ ¬W.
5. ¬P ∨ ¬Q ∨ R
6. U ∨ X
7. U ∨ ¬X
8. Q ∨ ¬U
9. ¬R ∨ ¬U

Initial valuation V0: All atoms unbound.

Sequence of calls.

I. Call dp1(ATOMS,S0,V0)
¬W is a pure literal. (W never appears) V1[W] = FALSE.
New set of clauses S1: Delete clauses 3 and 4 (satisfied)
1. P ∨ Q ∨ R
2. P ∨ ¬Q ∨ ¬R
5. ¬P ∨ ¬Q ∨ R
6. U ∨ X
7. U ∨ ¬X
8. Q ∨ ¬U
9. ¬R ∨ ¬U.

No pure literals, no singleton clauses.
Try V[P] := TRUE; V2 is the valuation V2[P] = TRUE, V2[W] = FALSE.
Call propagate(P,S1,V2): Delete clauses 1 and 2, delete ¬P from 5
New set of clauses S2:
5. ¬Q ∨ R
6. U ∨ X
7. U ∨ ¬X
8. Q ∨ ¬U
9. ¬R ∨ ¬U.

II. Call dp1(ATOMS, S2, V2).
No pure literals, no singleton clauses.
Try V[Q] := TRUE; V3 is the valuation V3[P] = TRUE, V3[Q] = TRUE, V3[W] = FALSE.
Call propagate(Q,S2,V3): Delete clause 8, delete ¬Q from 5
New set of clauses S3:
5. R
6. U ∨ X
7. U ∨ ¬X
9. ¬R ∨ ¬U.

III. Call dp1(ATOMS, S3, V3).
5 is a singleton clause with literal R;
V[R] = TRUE;

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V4 is the valuation V4[P]=TRUE, V4[Q]=TRUE, V4[R]=TRUE, V4[W]=FALSE.
Call propagate(R,S3,V4): Delete clause 5, delete ¬R from clause 9.
New set of clauses S4:
    6. U ∨ X
    7. U ∨ ¬X
    9. ¬U.

9 is a singleton clause with literal ¬U;
V[U] = FALSE;
V5 is the valuation V5[P]=TRUE, V5[Q]=TRUE, V5[R]=TRUE, V5[U]=FALSE, V5[W]=FALSE.
Call propagate(U,S4,V5): Delete clause 9, delete U from clauses 6 and 7.
New set of clauses S5:
    6. X
    7. ¬X

6 is a singleton clause with literal X;
V[X] = TRUE;
V6 is the valuation V6[P]=TRUE, V6[Q]=TRUE, V6[R]=TRUE,
              V6[U]=FALSE, V6[X]=TRUE, V6[W]=FALSE.
Call propagate(X,S5,V6): Delete clause 6, delete ¬X from clause 7.
New set of clauses S6:
    7. empty

7 is the empty clause.
III returns NIL to II.

II continuing.
Try V[Q] := FALSE; V7 is the valuation V7[P] = TRUE, V7[Q] = FALSE, V7[W] = FALSE.
Call propagate(Q,S2,V7): Delete clause 5, delete Q from 8
New set of clauses S7:
    6. U ∨ X
    7. U ∨ ¬X
    8. ¬U
    9. ¬R ∨ ¬U.

IV. Call dp1(ATOMS, S7, V7).
    8 is a singleton clause with literal ¬U;
    V[U] = FALSE;
    V8 is the valuation V8[P]=TRUE, V8[Q]=FALSE, V8[U]=FALSE, V8[W]=FALSE.
    Call propagate(U,S7,V8): Delete clauses 8 and 9, delete U from clauses 6 and 7.
    New set of clauses S9:
        6. X
        7. ¬X

    6 is a singleton clause with literal X;
    V[X] = TRUE;
    V9 is the valuation V9[P]=TRUE, V9[Q]=FALSE, V9[U]=FALSE,
              V9[X] = TRUE, V9[W]=FALSE.
    Call propagate(X,S8,V9): Delete clause 6 , delete ¬X from clause 7.
    New set of clauses S9:
        7. empty

7 is the empty clause.
IV returns NIL to II.

II having failed with both TRUE and FALSE for Q, returns NIL to I.

I continuing
Try V[P] := FALSE; V10 is the valuation V10[P] = FALSE, V10[W] = FALSE.
Call propagate(P,S1,V10): Delete clause 5, delete P from 1 and 2
New set of clauses S10:
    1. Q ∨ R
    2. ¬Q ∨ ¬R
    6. U ∨ X
    7. U ∨ ¬X
    8. Q ∨ ¬U
    9. ¬R ∨ ¬U.

V. Call dp1(ATOMS, S10, V10).
    No pure literals, no singleton clauses.
    Try V[Q] := TRUE; V11 is the valuation V11[P] = FALSE, V11[Q] = TRUE, V11[W] = FALSE.
    Call propagate(Q,S10,V11): Delete clauses 1 and 8, delete ¬Q from 2
    New set of clauses S11:
        2. ¬R
        6. U ∨ X
        7. U ∨ ¬X
        9. ¬R ∨ ¬U.

VI. Call dp1(ATOMS, S11, V11).
    ¬R is a pure literal.
    V[R] := FALSE; V12 is the valuation V12[P] = FALSE, V12[Q] = TRUE,
                                V12[R]=FALSE, V12[W] = FALSE.
    Delete clauses 2 and 9.
    New set of clauses S12:
        6. U ∨ X
        7. U ∨ ¬X
    U is a pure literal.
    V[U] := TRUE; V13 is the valuation V13[P] = FALSE, V13[Q] = TRUE, V13[R]=FALSE,
                                V13[U]=TRUE; V13[W] = FALSE.
    Delete clauses 6 and 7.
    S13 is the empty set of clauses.

    Set the value of atom X to be either TRUE or FALSE.
    Return V13 to the top level.