

Problem 3

This problem has to do with the programming assignment on Electric Charges in this repository:

<http://www.cs.nyu.edu/faculty/davise/MathTechniques/MoreAssigs/ProgElectric.pdf>

You should certainly think through this programming assignment before doing this problem, and since thinking through the programming assignment is practically equivalent to doing it, it is probably a good idea to complete that assignment before doing this problem.

The programming assignment is written in terms of charges along a plane, but the analysis of charges distributed in 3-dimensional space, or for that matter in d -dimensional space for any d is almost identical. The code that you have written for that assignment should apply either with no changes at all or with at most very small changes to space of arbitrary dimension.

Consider a case where k fixed charges are distributed in d -dimensional space and where a test charge is being placed and the forces are measured at w different locations. As in the programming assignment, you may assume that the locations are all known, and all that is unknown is the charges on the k objects.

A. The force measurements generate a system of linear equations. How many equations in how many unknowns? Your answer should be expressed in terms of d , w , and k .

B. Suppose that you are working in d -dimensional space, and there are k fixed charges that you want to determine. In general, how many placements of the test charge will be needed to uniquely determine all the fixed charges?

C. Construct an example where there are two charges in the plane and one placement of the test charge, but the measurement does not determine the values of the two charges uniquely.

D. Suppose now that each of the fixed charges is moving with a constant, known velocity. For instance, it is known that charge B_1 is moving along the trajectory $\mathbf{p}_1(t) = \langle 1, 1, 1 \rangle + t \cdot \langle 0, 2, 3 \rangle$ and likewise for the other B_i . When you place the test charge, you now record the time of the placement in addition to its position and the force. How does that affect your analysis for part (A)? (In actual fact, a moving charge also generates a magnetic field, but (a) you may assume that the speed is small enough that this can be ignored; (b) it doesn't change the answer to this question, because the magnetic field is also a linear function of the magnitude of the charge.)

E. Suppose finally that each of the fixed charges is moving with a constant, unknown velocity, though the position at time 0 is known. For instance, it is known that charge B_1 is moving along the trajectory $\mathbf{p}_1(t) = \langle 1, 1, 1 \rangle + t \cdot \vec{v}_1$. where \vec{v}_1 is unknown. As in part (D) you record the time of each placement. How does this new version of the problem affect the analysis in part (A)?