Is It Really Zero?

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Abstract

The *Zero Problem* asks whether a given numerical expression is equal to zero. Here are some numerical expressions (all zero):

$$1-1+1-1+1-1, \quad 3^2+4^2-5^2, \quad 1-\sum_{n=1}^{\infty} 2^{-n}, \quad \sqrt{2}+\sqrt{3}-\sqrt{5+2\sqrt{6}}, \quad etc.$$

Computer scientists call this a *decision problem* because we only have to decide between one of two possible answers. There is a different Zero Problem for each clearly defined class of numerical expressions. For instance, the last expression might be an instance of the class of *radical expressions* which involve integers, the four arithmetic operations and square-roots.

One might say: if we can approximate each operation in the expression to sufficiently high precision, then we can surely discover the answer. A little thought shows the inadequacy of this approach.

In this talk, we show hope to achieve three goals:

- Show why Zero Problems are at the heart of some difficult computational problems in many fields, from computational geometry to transcendental number theory.
- Show how to solve the Zero Problem in the algebraic case.
- Give a software demonstration of our `Core Library`, a software package in `C++` language. You will see how the ability to decide zero in our library can greatly simplify the programmer's task of achieving robust geometric algorithms.

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