FUNDAMENTAL ALGORITHMS OLD MIDTERM

Maximum Score 140. Do all problems.

1. (20) Give an algorithm HORSE with the following property. The input is two arrays \( A[1 \cdots N], B[1 \cdots N] \), both arrays in increasing order. The output is an array \( C[1 \cdots (2N)] \) which has all the values of the arrays \( A, B \) and is in increasing order. How long does your algorithm take? (Brief reason please!)

2. (20) Illustrate the operation of COUNTINGSORT on the array

\[
A = (2, 1, 2, 1, 0, 0, 0, 1)
\]

with \( n = 8 \) and \( k = 2 \). Pictures and some well chosen words, please. (You do not need every detailed step but you must make clear the main steps.)

3. (20) For the following algorithms let \( T(N) \) denote the total number of times the step after the WHILE step is reached. For the first algorithm give (five points) an exact formula for \( T(N) \). For the second algorithm first (ten points) give \( T(N) \) as a precise sum. Then (five points) Find \( T(N) \) is the form \( T(N) = \Theta(g(N)) \) for a standard \( g(N) \). Reasons please!

   (a) \( X=1 \)
       
       WHILE \( X \lt N \)
       
       do \( X=2^X \)

   (b) FOR \( I=1 \) TO \( N \)
       
       \( X=1 \)
       
       WHILE \( X^2 \le I \)
       
       \( X++ \)

4. (10) In hashing, what are collisions? Describe one method (your choice!) for dealing with them.

5. (20) Let \( A \) is a max-heap with heapsize \( N \). Describe a program called here BIGGULP(\( A, i \),key) that replaces \( A[i] \) by a value \( key \) which is bigger than \( A[i] \) and then restores the heap property. How long does BIGGULP take? How long does BIGGULP take in the special case when \( i = 1 \)?
6. (15) You want to sort five elements $a, b, c, d, e$ using seven paired comparisons. Assume that your question is “Is $a < b$” and that the answer was Yes. Assume that your second question is “Is $a < c$.” Using the Information-Theoretic Lower Bound prove that you will not be able to sort the elements.

7. (15) There is an algorithm $\text{RABBIT}(A, B)$ that multiplies two $n \times n$ matrices $A, B$ by performing seven multiplications of $(n/2) \times (n/2)$ matrices and then performing $O(n^2)$ further operations. Create a recursive equation for the time $T(n)$ that $\text{RABBIT}(A, B)$ takes and use the Master Theorem to give $T(n)$ asymptotically.

8. (15) Let $A[1\cdots N]$ be an array with all entries integers between 0 and $N$. How long would $\text{RADIX-SORT}$ take to sort $A$ assuming that we use base 2 (that is, binary)? (Assume the entries $A[I]$ are already given as binary strings in the input.) You must give an argument for your answer.

9. (5) State the binary-search-tree property. (That is, the condition that the keys are required to fulfill.)