I wasn’t sure anymore and I will tell you, it is a strange process to feel one’s mind changing, allowing ideas into your brain which it had once considered unthinkable. I cannot say it’s painful, or particularly pleasurable, but that it requires a certain relaxation of the hold one keeps over oneself, and is to that degree both a thrill and a horror.
– from The Chess Garden, by Brooks Hansen

1. Some exercises in which \( n \) is NOT the data size but we want the answer in terms of \( n \). (Answers in \( \Theta \)-land.)

   (a) How long does \textsc{merge-sort} on \( n^2 \) items take?
   (b) Suppose that when \( n = 2^m \), \textsc{anna} takes time \( \Theta(m^22^m) \). How long does it take as a function of \( n \).
   (c) Suppose that when \( n = 2^m \), \textsc{bob} takes time \( \Theta(5^m) \). How long does it take as a function of \( n \).
   (d) How long does \textsc{counting-sort} take to sort \( n^2 \) items with each item in the range 0 to \( n^3 - 1 \).
   (e) How long does \textsc{radix-sort} take to sort \( n^2 \) items with each item in the range 0 to \( n^3 - 1 \) and base \( n \) is used.

2. Consider hashing with chaining using as hash function the sum of the numerical values of the letters (A=1,B=2,...,Z=26) mod 7. For example, \( h(\text{JOE}) = 10 + 15 + 5 \mod 7 = 2 \). Starting with an empty table apply the following operations. Show the state of the hash table after each one. (In the case of Search tell what places were examined and in what order.)
   Insert COBB
   Insert RUTH
   Insert ROSE
   Search BUZ
   Insert DOC
   Delete COBB

3. Wally Wonkle, NYU drone, is asked to create a hash table (by chaining) of the 20000 NYU student records. Each record takes about 10K bytes of memory. He decides to use a hash table of size one million and
The Boss goes ballistic, accusing him of negligent use of precious computer space. Argue coherently that this method is not using very much more space than a hash table of size 20000. What is the advantage in using size one million instead of size 20000?

4. Consider a Binary Search Tree $T$ with vertices $a, b, c, d, e, f, g, h$ and $ROOT[T] = a$ and with the following values ($N$ means NIL)

<table>
<thead>
<tr>
<th>vertex</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>parent</td>
<td>N</td>
<td>e</td>
<td>e</td>
<td>a</td>
<td>d</td>
<td>g</td>
<td>c</td>
<td>a</td>
</tr>
<tr>
<td>left</td>
<td>h</td>
<td>N</td>
<td>N</td>
<td>e</td>
<td>c</td>
<td>N</td>
<td>f</td>
<td>N</td>
</tr>
<tr>
<td>right</td>
<td>d</td>
<td>N</td>
<td>g</td>
<td>N</td>
<td>b</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>key</td>
<td>80</td>
<td>170</td>
<td>140</td>
<td>200</td>
<td>150</td>
<td>143</td>
<td>148</td>
<td>70</td>
</tr>
</tbody>
</table>

Draw a nice picture of the tree. Illustrate $\text{INSERT}[i]$ where $\text{key}[i] = 100$.

If you want to have good ideas you must have many ideas. Most of them will be wrong, and what you have to learn is which ones to throw away.
– Linus Pauling