Sandwiched as we are between the “everything” that is behind us and the “zero” beyond us, ours is an ephemeral existence in which there is neither coincidence nor possibility. Haruki Murakami, A Wild Sheep Chase

1. In $S_3$ (reminder, this is our standard notation for the permutations on $\{1, 2, 3\}$) show that there are four elements $x$ satisfying $x^2 = e$ and three elements $x$ satisfying $x^3 = e$.

2. In $\mathbb{Z}_{13}^*$ let $H = \{1, 5, 12, 8\}$. List the right cosets $Ha$.

3. Let $G$ be the symmetries of the square. (See the solutions to assignment 1 for a table.) Let $H = \{I, V\}$. List the right cosets $Ha$ and the left cosets $aH$. Do the same with $H = \{I, R, S, T\}$.

4. Let $G = S_n$ and let $H = \{\sigma \in G : \sigma(1) = 1\}$. Let $\tau \in G$ and suppose $\tau(i) = 1$. By $\tau H \tau^{-1}$ we mean all elements of the form $\tau \sigma \tau^{-1}$ where $\sigma \in H$.

(a) Show that if $\gamma \in \tau H \tau^{-1}$ then $\gamma(i) = i$

(b) Show that if $\gamma(i) = i$ then $\gamma \in \tau H \tau^{-1}$
5. Give the elements and the multiplication table for $\mathbb{Z}^*_15$. (The elements of $\mathbb{Z}^*_n$ are those $i$, $1 \leq i \leq n-1$ which are relatively prime\(^1\) to $n$. Multiplication is modulo $n$.) Find the order $o(a)$ of each element $a$. (The order of $a$ is the least positive integer $n$ so that $a^n = 1$.)

6. The center $Z$ of a group $G$ is the set of all $z \in G$ with the property that $zg = gz$ for all $g \in G$. Prove that $Z$ is a subgroup of $G$. Prove the $Z$ is Abelian.

I cannot pretend I am without fear. But my predominant feeling is one of gratitude. I have loved and been loved; I have been given much and I have given something in return; I have read and traveled and thought and written. I have had an intercourse with the world, the special intercourse of writers and readers.

Above all, I have been a sentient being, a thinking animal, on this beautiful planet, and that in itself has been an enormous privilege and adventure.

Oliver Sacks, 1933-2015

\(^1\)Positive integers $m, n$ are called relatively prime if they have no common factor. Do not confuse this with primality!