An equation means nothing to me unless it expresses a thought of God. – Ramanujan
You don’t have to believe in God but you should believe in The Book. – Paul Erdos

Notation: In an I.D. $D$ we let $(a, b)$ denote the ideal \{ra + sb : r, s \in D\}.

1. Call $R$ a Turkey-Yam Ring if it is an Integral Domain and so that it has a “size function” $d : R - \{0\} \to \{0, 1, 2, \ldots\}$ with $d(\alpha) \leq d(\alpha\beta)$ and with the following property: For all $\alpha, \beta \in R - \{0\}$ with $\beta$ not dividing $\alpha$ there exist $m, n \in R$ with $m\beta + n\alpha \neq 0$ and $d(m\beta + n\alpha) < d(\beta)$. Prove that a Turkey-Yam Ring is a Principal Ideal Domain. Idea: Modify the proof that a Euclidean Domain is a Principal Ideal Domain.

2. Give the Prime Factorization in $\mathbb{Z}[i]$ of: $6, 10 + 3i, 4 + 7i$.

3. JUST FOR FUN:
   (a) Name this Paul Simon song (websearch OK!): $\forall_x \exists_y[x] = [y]
   (b) Simplify: $(x - a)(x - b) \cdots (x - z)$
   (c) Find $\int \frac{d(cabin)}{cabin}$
   (d) If $\lim_{x \to 8} \frac{1}{x - 8} = \infty$ find $\lim_{x \to 5} \frac{1}{x - 5}$.

4. Let $a, b$ be relatively prime integers and nonzero. Set $M = (a + bi, a - bi)$ in $\mathbb{Z}[i]$.
   (a) Prove $2a \in M$ and $2b \in M$. Then prove $2 \in M$.
   (b) List all ideals $N \subset \mathbb{Z}[i]$ with $2 \in N$. Give a reason!
   (c) Prove $M \neq (2)$. (Idea: $a + bi \in M$.)
   (d) Find, with proof, $\gcd(a + bi, a - bi)$ when $a$ is even and $b$ is odd.
   (e) Find, with proof, $\gcd(a + bi, a - bi)$ when $a$ is odd and $b$ is even.
   (f) Find, with proof, $\gcd(a + bi, a - bi)$ when $a$ is odd and $b$ is odd.

5. Apply the Euclidean Algorithm on $\mathbb{Z}[i]$ to find $\xi = \gcd(3 + 5i, 7 - 5i)$

I cannot live without people. – Pope Francis