Honors Algebra I, Assignment 1
Not to Be Submitted

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Course website (bookmark!):
www.cs.nyu.edu/cs/faculty/spencer/algebra/index.html
or go to Prof. Spencer’s home page and look for pointer.

1. List with description the symmetries of the square. (There are eight of them.) Give the table for the products of the symmetries. Give the inverse of each symmetry.

2. Let $G$ denote the set of linear functions $f(x) = mx + b$ on the real line with $m \neq 0$. Denote such a function by $(m, b)$. Define a product $f^*g$ as the function $h(x) = g(f(x))$. For $f(x) = 3x + 7$ and $g(x) = 5x - 2$ what are $f^*g$ and $g^*f$? More generally, for $f = (m_1, b_1)$ and $g = (m_2, b_2)$ find $f^*g$ and $g^*f$. For $f = (m, b)$ find a formula for that $g$ so that $f^*g = (1, 0)$.

3. Let $GL_n(R)$ denote (this is standard) the general linear group over $R$. That is, the elements are the $n \times n$ nonsingular matrices $A$ and the group operation is matrix multiplication. Let $H$ denote those $A \in GL_n(R)$ with positive determinant. Prove that $H$ is a subgroup of $GL_n(R)$.

   In general, given a group $G$ (here $GL_n(R)$) and a nonempty subset $H \subset G$, to prove that $H$ is a subgroup of $G$ you need to show the following three things

   (a) **Identity**: The identity element $I \in H$.

   (b) **Multiplicative Closure**: If $A, B \in H$ then $AB \in H$

   (c) **Inverse Closure**: If $A \in H$ then $A^{-1} \in H$.

   (Note: In other situations the identity element may have different names, such as $0, e, 1$. Also, if the operation is written as addition then $AB$ becomes $A + B$ and $A^{-1}$ becomes $-A$.)

   Beauty is the first test: there is no permanent place in the world for ugly mathematics. – G.H. Hardy