Homework 5
Scientific Computing

Assigned: Wed Nov 2 Due: Wed Nov 9

Go to **www.ampl.com**, click on **Try AMPL!**, accept the conditions, select **diet.mod** and **diet.dat** from the list of possible **model files** and **data files**, and click **Submit**. More in a moment, but first let me explain...

The **diet problem** is a classical example of LP. It's appealing not because it is a practical example, but because it is easy to understand, fun, and it's not hard to see how more practical problems might be expressed in such a framework. The idea of the diet problem is that a person is choosing a diet from a set of possible foods, with constraints on the nutrients included in the diet, as well as on the amounts of each food that can be consumed. The objective is to minimize the cost. In summary, the diet problem can be written in the form

$$\begin{array}{rl} \min & c^T x \\ \text{subject to} & g \leq Ax \leq h, \quad \ell \leq x \leq u \end{array}$$

Here the (i, j) entry of A, say a_{ij}, is the known amount of nutrient i in food j, and x_j is the unknown variable: the amount of food j to be in the diet. This must be between ℓ_j and u_j. On the other hand, summing $a_{ij}x_j$ tells you the total amount of nutrient i in the diet; this must be between g_i and h_i.

Going back to the web page: after selecting **diet.mod** and **diet.dat** and clicking **Submit**, carefully study the information in the window that is displayed. You see that a bunch of parameters (cost, amt, and so on) are declared. The **cost** vector is what we called c above, and the **amt** matrix is A. The quantity **amt**{NUTR,FOOD}, or in matrix notation a_{ij}, is the amount of nutrient NUTR in food FOOD. The other parameters are lower and upper bounds on the amount allowed of each food, and lower and upper bounds on the required total nutrients. Scrolling down, you see that **data** values are assigned to these parameters below (these come from the **diet.dat** file). Before assigning the numerical values, the NUTR and FOOD index sets are defined (NUTR: Vitamins A, B1, B2 and C; FOOD: Beef, Chicken, Fish, Ham, Macaroni and Cheese (!), Meat Loaf (!), Spaghetti and Turkey). Don't go any further until you understand the notation.

Now, turn your attention to the **Solver** menu. This selects between various software packages that can be used to solve the LP that is defined in the window. The default is MINOS, a top-quality Fortran package from Stanford. Not all these solvers are actually relevant for solving LP's!

Using Minos, click **Send**. This solves the LP and you will see that the optimal solution is a diet consisting entirely of Macaroni and Cheese!

Questions to answer:

1. Obviously, all the variables are on their lower bounds of 0 except MCH. We say that all lower bounds except the one for MCH are *active*. But what about the nutrient constraints? Which of these are on their lower or upper bounds? In other words, which of these constraints have active lower or upper bounds? To answer this you either have to do some computation (e.g., by pasting the data into Matlab or Excel), or (better) figure out how to display what you want in AMPL by adding to the display directive in the middle window.

2. We say that an optimal solution to an LP of the form on the previous page is *basic* if the total number of *active* bounds on the variables and constraints is at least equal to the number of variables. In the light of your answer to the previous question, is the displayed optimal solution basic?

3. A second diet data file **diet2.dat** attempts to define a more healthy diet. Select this instead of **diet.dat** (if your Back button doesn't work, you can start over; you may want to bookmark the page). Compare this data file with the first and observe the differences. Now click **Send** to solve this new problem. You see the LP is infeasible - too many constraints have been added. Relax some of the constraints in the new problem sufficiently (using a trial and error process) until you have an LP which is feasible and for which the solution is more interesting than the first problem. Print the web page, making sure both input and output are visible, and explain whether the optimal point displayed is basic.

4. Get creative and introduce some new foods. You don't need to use real data. While you are at it, you may as well overhaul the choice of foods completely, using a less American (and more healthy?) cultural norm! The point of this question is to get you to play with the model, being careful to respect the AMPL syntax, and come up with a more interesting diet. Print the resulting web page, making sure that all the

input and output is visible. Again, explain whether the optimal point is basic.

5. Finally, solve the same LP (the one you made up) using any other LP solver you have available. The most convenient is probably Matlab's **linprog**, which accepts problems in a slightly different form; you will have to think about how you can convert your problem to fit its format (type **help linprog** for details on the format and parameter list). There is more than one valid way to do this.

AMPL stands for A Mathematical Programming Language. There is a copy of the AMPL book in the CIMS library. I don't think you need to consult this, but in case you wish to do so, I am putting it on reserve. There is also one more(in fact the latest version) at Bobst which is not on reserve and so available to the first enterprising person who wants to take it out!