Dexterous Manipulation: A Geometric Approach

(A Survey Paper)

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[1]

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Abstract

These notes present in a self-contained manner the basic facts about the theory of dexterous manipulation. In particular, we discuss the models of robot hands and analysis and synthesis of multifingered grasps for objects of arbitrary geometry. The techniques exploit an interesting connection between the problems of dexterous manipulation and some classical results in combinatorial geometry.

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1. Introduction

The area of dexterous manipulation studies the problems of grasping, fine-manipulation and control with multi-fingered robot hands (see [1, 5, 14, 15, 19]. These hands usually consist of a small number (four or five) of articulated fingers, possibly with a palmar surface. A variety of finger arrangements have been suggested, the more common being an anthropomorphic arrangement, which consists of an opposable 'thumb' and the others arranged to work cooperatively.

The science and technology of dexterous manipulation, however, are still in their infancy. Many of the grasping and control algorithms for manipulation implicitly assume fairly well-controlled and well-modeled environments: for instance, the algorithms developed by [20] is strongly dependent on availability of gravitational force. In an attempt to develop schemes that work in more general situations, researchers have begun to study the underlying mathematics; these results are somewhat theoretical in nature and assume a rather idealized hand. Here, we shall attempt to survey the recent progress and pose some of the open problems.

2. Grasping: Terminology and Theory

Consider an idealized robot hand, consisting of several independently movable force-sensing fingers; this hand is used to grasp a rigid object B. Furthermore, we make the following simplifying assumptions:

- (*Smooth Body*) B is a full-bodied (i.e. no internal holes) compact subset of the Euclidean 3-space. Furthermore, B has a piece-wise smooth boundary ∂B.

- (*Point Contact*) For each finger-contact on the body, we may associate a nominal point of contact, $\mathbf{p} \in \partial B$. We let ∂B^* denote the set of points $\mathbf{p} \in \partial B$ such that the direction $\mathbf{n}(\mathbf{p})$ normal to ∂B at $\mathbf{p}$ is well-defined; by convention, we pick $\mathbf{n}(\mathbf{p})$ to be the unit normal pointing into the interior of B.

  For each such point $\mathbf{p}$, we can define a wrench system $\{\Gamma^{(1)}(\mathbf{p}), \Gamma^{(2)}(\mathbf{p}), \ldots, \Gamma^{(m)}(\mathbf{p})\}$, $(0 \leq m \leq 6)$, where the number and screw-axes of the wrench system depend on the contact type. Some of these wrenches can be *bisense* (i.e. can act in either sense) and the remaining wrenches, *unisense*. (For a discussion of screw theory, and in particular, wrenches and twists, see [4] and [13].)

- (*Compliance*) We will consider the case when the fingers are stiff—the force/torques applied at the fingers are generated by some actuators whose mechanics need not concern us.

Many interesting special cases occur, depending on how we model the *static friction* and the *stiction* between the fingers and the body B. In the case, where the contacts are frictionless, a finger can only apply force $\mathbf{f}$ on the body in the direction $\mathbf{n}(\mathbf{p})$ at the point $\mathbf{p}$. Also if the fingers are non-sticky, then the force $\mathbf{f}$ has a non-negative magnitude, $f = \mathbf{f} \cdot \mathbf{n}(\mathbf{p}) \geq 0$. Such grips are also known as 'positive grips'. In this case, the wrench system associated with each point is:

$$\Gamma(\mathbf{p}) = \{[\mathbf{n}(\mathbf{p}), \mathbf{p} \times \mathbf{n}(\mathbf{p})]\}$$

Thus, corresponding to a set of finger-contacts, we have a system of n wrenches,

$$\{\mathbf{w}_1, \ldots, \mathbf{w}_k, \mathbf{w}_{k+1}, \ldots, \mathbf{w}_n\},$$

the first k of which are bisense and the remaining last $n-k$ of the wrenches are unisense. Let us assume that the magnitudes of these wrenches are given by the scalars f_i's

$$\{f_1, \ldots, f_k, f_{k+1}, \ldots, f_n\},$$

where f_1, ..., $f_k \in \mathbb{R}$ and f_{k+1}, ..., $f_n \in \mathbb{R}_+$, and not all the magnitudes are zero. We call such a system of wrenches and the wrench-magnitudes, a *grip*, G, and say that this grip G generates an external wrench $\mathbf{w} = [F_x, F_y, F_z, \tau_x, \tau_y, \tau_z] \in \mathbb{R}^6$, if

$$\mathbf{w} = \sum_{i=1}^{n} f_i\, \mathbf{w}_i.$$

Next, we consider the concept of a *closure grasp*: A system of wrenches $\mathbf{w}_1$, ..., $\mathbf{w}_n$ (as before) is said to constitute a *force/torque closure grasp* if and only if any arbitrary external wrench can be generated by varying the magnitudes of the wrenches (subject to the constraints imposed by the senses of the wrenches). A necessary and sufficient condition for a closure grasp is that the (module) sum of the linear space spanned by the vectors $\mathbf{w}_1$, ..., $\mathbf{w}_k$ and the positive space spanned by the vectors $\mathbf{w}_{k+1}$, ..., $\mathbf{w}_n$ is the entire $\mathbb{R}^6$:

$$\text{lin}\ (\mathbf{w}_1, \ldots, \mathbf{w}_k) + \text{pos}\ (\mathbf{w}_{k+1}, \ldots, \mathbf{w}_n) = \mathbb{R}^6.$$

Let us denote, by L, the linear space lin ($\mathbf{w}_1$, ..., $\mathbf{w}_k$), and, by $L^\perp$, the orthogonal complement of L in $\mathbb{R}^6$. Let π be the linear projection function of $\mathbb{R}^6$ onto $L^\perp$ whose kernel is L. Then it can be shown that a necessary and sufficient condition for a closure grasp is

$$\text{lin}\ (\mathbf{w}_1, \ldots, \mathbf{w}_k) + \text{pos}\ (\pi\mathbf{w}_{k+1}, \ldots, \pi\mathbf{w}_n) = \mathbb{R}^6.$$

The above equation in turn is equivalent to the following conditions:

$$\mathbf{0} \in \text{int conv}\ (\pi\mathbf{w}_{k+1}, \ldots, \pi\mathbf{w}_n)$$

in $L^{\perp}$. Here, if $k = 0$ (i.e. positive grip) then the above condition reduces to the following:

$$\mathbf{0} \in \text{int conv } (\mathbf{w}_1, \ldots, \mathbf{w}_n).$$

Let us assume that $\dim(L) = d$. Then there is a linear basis W of L

$$W = \{\mathbf{w}_{j_1}, \ldots, \mathbf{w}_{j_d}\} \subseteq \{\mathbf{w}_1, \ldots, \mathbf{w}_k\}$$

which when adjoined with a set of vectors W',

$$W' = \left\{\mathbf{w}_{j_{d+1}}, \ldots, \mathbf{w}_{j_6}\right\} \subseteq \{\mathbf{w}_{k+1}, \ldots, \mathbf{w}_n\}$$

yields $\widehat{W} = W \cup W'$, a linear basis of $\mathbb{R}^6$. Thus under the condition that we have a closure grasp, we can find $\mathbf{g} \in \mathbb{R}^k \times \mathbb{R}_+^{n-k}$ such that

$$-\left(\mathbf{w}_{j_{d+1}} + \cdots + \mathbf{w}_{j_6}\right) = \sum_{i=1}^{n} g_i \; \mathbf{w}_i,$$

and thus

$$\sum_{i=1}^{n} f_{h,i} \; \mathbf{w}_i, \quad = \quad \mathbf{0}, \quad \text{where } \mathbf{f}_h \in \mathbb{R}^k \times \mathbb{R}_+^{n-k}, \text{ and } f_{h,j_{d+1}} > 0, \ldots, f_{h,j_6} > 0.$$

Now any external wrench $\mathbf{w}$ can be expressed as a linear combination of the vectors in the basis $\widehat{W}$. Thus there is a vector $\mathbf{f}_p \in \mathbb{R}^n$, whose non-zero entries are in the positions j_1, ..., j_6, and

$$\mathbf{w} = \sum_{k=1}^{6} f_{p,j_k} \; \mathbf{w}_{j_k} = \sum_{i=1}^{n} f_{p,i} \; \mathbf{w}_i.$$

Now consider a vector $\mathbf{f} = \mathbf{f}_p + \lambda \; \mathbf{f}_h \in \mathbb{R}^k \times \mathbb{R}_+^{n-k}$, where λ is chosen to be of a sufficiently large positive value that ensures that the negative components in $\mathbf{f}_p$ are dominated by the positive components of $\mathbf{f}_h$. Thus,

$$\mathbf{w} = \sum_{i=1}^{n} f_i \; \mathbf{w}_i = \sum_{i=1}^{n} (f_{p,i} + \lambda \cdot f_{h,i}) \; \mathbf{w}_i.$$

These arguments yield a simple algorithm to find *at least* one set of force targets that can generate a given external wrench. Also, as the external wrench is varied in the course of a manipulation task, this algorithm updates the force targets fast.

At this point, we emphasize that the above formulation has turned a problem in mechanics into a purely geometric problem, now amenable to many interesting techniques in convexity theory and computational and combinatorial geometry. This observation has been most fruitfully exploited in the work of [11] and [12].

3. The Case of a Positive Grip

In order to obtain a particular grasp on an object, it must be determined if that grasp is achievable. It is for this reason, researchers have studied the question of how many fingers (wrenches) are required to obtain certain grasps on the object.

Reuleaux and Somoff determined that the closure grasp of a two dimensional object requires at least *four* wrenches and of a three dimensional object requires at least *seven*, where the wrenches are normal to the surface of the object.

Mishra, Schwartz, and Sharir [11] gave the most general bounds on the number of fingers in the case of a *positive grip*; they also provided an algorithm that finds *at least one* such grip on a polyhedral object and their algorithm runs in time linear in the number of faces of the object. Similar results have also been obtained by Markenscoff, Ni, and Papadimitriou [9], although their proof techniques are quite different from those used by [11]. Very recently, Meyer [10] has improved the bound on the number of fingers to *seven*, under the assumption that the object to be grasped is convex polyhedral. In this section, we briefly describe the techniques of Mishra, Schwartz, and Sharir.

Recall that a non-frictional grip is called a *positive grip*. Note that, in this case, the fingers are assumed to be point fingers, a finger can only apply a force on the object along the surface-normal at the point of contact, directed inward. In this situation, we have a *wrench map*, Γ, mapping ∂B^* into the six-dimensional *wrench space* $\mathbb{R}^6$ as follows:

$$\Gamma\colon \partial B^* \to \mathbb{R}^6 : \mathbf{p} \mapsto [\mathbf{n}(\mathbf{p}), \mathbf{p} \times \mathbf{n}(\mathbf{p})].$$

Essentially, Γ maps $\mathbf{p}$ to the point $\Gamma(\mathbf{p})$ in the wrench space that represents the effects of applying a unit force at $\mathbf{p}$ in the direction $\mathbf{n}(\mathbf{p})$.

Now, given an arbitrary compact rigid object B whose piece-wise smooth surface ∂B is not a "surface of revolution," it can be shown that

> **B can be held with a closure grasp by a positive grip of at most twelve fingers.**

The assumption that ∂B is not a surface of revolution is essential, since, in the absence of friction, no matter how many fingers are employed the object can always be rotated about its axis of symmetry.

By virtue of the discussion of the previous section, we only need to show that

$$\Big(\exists\ \mathbf{p}_1, \ldots, \mathbf{p}_k \in \partial B^*,\ k \le 12\Big)\ \Big[\mathbf{0} \in \text{int conv}\ \Big(\Gamma(\mathbf{p}_1), \ldots, \Gamma(\mathbf{p}_k)\Big)\Big].$$

The proof is in two steps:

(1) We claim that

$$\mathbf{0} \in \text{int conv}\ \Gamma(\partial B^*).$$

Using Gauss' divergence theorem, we see that

$$\left[\int_{\partial B^*} \mathbf{n}(\mathbf{p})\, dS,\ \int_{\partial B^*} (\mathbf{p} \times \mathbf{n}(\mathbf{p}))\, dS\right] = \mathbf{0},$$

i.e.

$$\mathbf{0} \in \text{conv } \Gamma(\partial B^*).$$

Thus, it remains to show that the origin is indeed an interior point of the convex hull; assume to the contrary. Then it can be shown that there is a nonzero vector $\mathbf{g} = [F, \tau] \in \mathbb{R}^6$ orthogonal to the linear subspace spanned by $\Gamma(\partial B^*)$. Thus, for each function $c(\mathbf{p})$ we have

$$\int_{\partial B^*} c(\mathbf{p}) \; [(F + \tau \times \mathbf{p}) \cdot \mathbf{n}(\mathbf{p})] \; dS = 0.$$

In particular, substitute the function $c(\mathbf{p}) = (F + \tau \times \mathbf{p}) \cdot \mathbf{n}(\mathbf{p})$ into the last equation, to deduce that $(F + \tau \times \mathbf{p}) \cdot \mathbf{n}(\mathbf{p})$ must be identically 0 over ∂B^*. But, this is possible only in the case when the surface of B is a surface of revolution.

(2) The rest follows by an application of the following theorem from combinatorial geometry:

> **Steinitz's Theorem.** [18]
>
> *If $X \subseteq \mathbb{R}$ and $\mathbf{p} \in$ int conv X, then $\mathbf{p} \in$ int conv Y for some $Y \subseteq X$ with $|Y| \leq 2d$.*

Our main result then follows from Steinitz's Theorem, once we identify X with $\Gamma(\partial B^*)$, and Y with the set $\{\Gamma(\mathbf{p}_1), \ldots, \Gamma(\mathbf{p}_k)\}$, with $k \leq 2 \times 6 = 12$.

4. Efficiency of a Closure Grasp

The techniques of the previous sections yield a method to synthesize at least *one* closure grasp of an object. However, in the absence of any measure of "goodness" for closures grasps, the synthesized grasp may not be very robust, thus, not useful in practice. This has motivated further research that attempts to quantify the goodness of closure grasps and to synthesize provably good closure grasps [6].

One criterion for goodness is the "efficiency" of a grasp, which roughly measures the amount of external force and torque that can be resisted by applying at most a unit of force at each grasp point. Kirkpatrick, Mishra and Yap [6] have shown that this efficiency metric can be derived from a stronger *quantitative version* of the Steinitz's theorem. Before we explore this connection, we need to introduce few notations, as follows.

For any set $X \subseteq \mathbb{R}$, let the *residual ball* of X refer to the maximal ball $B(X)$ centered at the origin $\mathbf{0}$ such that $B(X)$ is fully contained inside the convex hull of X. The *residual radius* of X, denoted $r(X)$ is the radius of this residual ball $B(X)$. Let

$$\begin{aligned} r_d(m, X) &= \max\{r(Y) \colon Y \subseteq X \text{ and } |Y| \leq m\}, \\ r_d(m) &= \min\{r_d(m, X) \colon X \subseteq \mathbb{R} \text{ and } \leftthreetimes(\mathbb{X}) \geq \nVdash\}. \end{aligned}$$

In the notation, we shall omit the subscript d, if the dimension is clear from the context. (In this paper, the interesting case is $d = 6$.) Thus the original Steinitz's theorem can now be interpreted to say that

$$r_d(2d) > 0.$$

A quantitative version of Steinitz's theorem provides more precise bounds for the number $r_d(m)$, when $m \geq 2d$.

Now, the achievable efficiency of a closure grasp with a m-fingered positive grip for a body B can seen to be the value, $r_6(m, \Gamma(\partial B^*))$. To see this, note that if we choose m points in $\Gamma(\partial B^*)$ with residual radius r then any wrench vector $\mathbf{v}$ whose Euclidean norm is at most r can be written as a convex combination of the m chosen points. So if $\mathbf{v}$ is any external wrench that is applied to the body B, and $\mathbf{v}$ lies in the residual ball of radius r, we can resist this external wrench by applying suitable forces (of magnitude at most 1) at the grasp points such that these forces sum to $-\mathbf{v}$; hence, we maintain the body in equilibrium. Thus, we see that the quantity $r_6(m)$ gives a universal measure for the efficiency of a closure grasp with $m \geq 12$ fingers. The special case, where $m = 2d$ had been studied by Bárány, Katchalski and Pach [2]; they showed that

$$r_d(2d) > \frac{c}{(2ed)^{\lfloor d/2 \rfloor} d^2}.$$

These results seem to indicate that a twelve-finger positive grip, while sufficient to provide a closure grasp, may not be adequate to achieve a desirable efficiency. Kirkpatrick, Mishra and Yap [6] have provided the following general bounds:

For any set $X \subseteq \mathbb{R}$ whose convex hull contains the unit ball B^d centered at the origin $\mathbf{0}$, we can find a set $Y \subseteq X$ of at most m points with residual radius $r(Y)$ of at least

$$1 - 3d \left(\frac{2d^2}{m} \right)^{\frac{2}{d-1}},$$

where m is assumed to be sufficiently large.

Furthermore, let $X \subseteq \mathbb{R}$ be the set of all points on the surface of the d-dimensional unit ball centered at the origin $\mathbf{0}$. Then, every subset $Y \subseteq X$ of at most m points has a residual radius $r(Y)$ of at most

$$1 - \frac{1}{13} \left(\frac{2d^2}{m} \right)^{\frac{2}{d-1}}.$$

In summary, for sufficiently large m ($m \geq 13^d d^{(d+3)/2}$)

$$1 - 3d \left(\frac{2d^2}{m} \right)^{\frac{2}{d-1}} \leq r_d(m) \leq 1 - \frac{1}{13} \left(\frac{2d^2}{m} \right)^{\frac{2}{d-1}}.$$

This indicates an interesting *trade-off* between efficiency of a closure grasp and the number of fingers. Thus, it is dubious whether closure grasps can be employed in practice to implement control algorithms for dexterous manipulation, where no regrasping is necessary during the entire task execution.

However, if one allows large number of fingers, or allows frictional and/or soft contact models, there is a possibility for synthesizing moderately efficient closure grasps. Kirkpatrick et. al. have provided certain approximation algorithms for these problems and some related computational geometric problems. However, still much research is needed to provide practical algorithms.

5. Future Research

So far, we have only discussed the static problems (i.e. grasping) in dexterous manipulation. There are still a large number of exciting and challenging questions related to the dynamic aspects of dexterous manipulation. Examples of such questions include fine-manipulation of objects by continuous finger-gaiting [3], dexterous adjustment of grasp configuration [7], prehension strategy and motion planning [16, 17] and controlled motion of grasped object [8].

A variety of issues involving high-level task planning remain largely unexplored. A thorough understanding of these questions are crucial to bring the science of dexterous manipulation into practice.

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