⟨3⟩ NON-OBLIVIOUS VERIFIERS

$\left\{ \begin{array}{l} \text{Function of } \hat{\mathcal{U}}_s(\cdot, v_j, v_j) \\ \quad [\text{General opinion w.r.t. item } v_j] \end{array} \right.$

$\alpha(u_i, V_j)$ = Pr. that u_i will reject all $v_j \in V_j$.

NETFLIX GAME MATRIX

$N = \{\perp \cup \mathbb{R}_+\}^{m \times n}$ $\quad m \times n$ Matrix

$$N_{i,j} = \begin{cases} \in \mathbb{R}_+ & \text{if } (u_i, v_j, v_j) \text{ is labeled} \\ \perp & \text{if unlabeled.} \end{cases}$$

$$\begin{array}{c} \\ u_1 \\ \vdots \\ \\ u_m \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & \cdots & & v_n \end{array} \\ \left\{ \begin{array}{cccc} \perp & \perp & \cdots & \perp \\ \perp & 2 & \cdots & 5 \\ 6 & 3 & \cdots & \perp \\ \vdots & \vdots & \ddots & \vdots \\ \perp & 4 & \cdots & \perp\perp \end{array} \right\} \end{array} = N \approx UDV^T = \hat{\mathcal{U}}_s$$

$U = \mathbb{R}^{m \times k}$

$V = \mathbb{R}^{n \times k}$

$D = \mathbb{R}^{k \times k} \rightarrow$ Diagonal

UDV^T = $m \times n$ matrix of rank k minimizes a loss-function.

NOTE BOOK

SINGULAR VALUE DECOMPOSITION

N = $m \times n$ matrix

m points in n-dimensional space.

Sender View } (1) Each point represents a user
(2) The row vector $\in \mathbb{R}^n$ is his utility function.

Receiver View } There is a different matrix
$$P \in \mathbb{R}^{n \times m}$$
which represents the receiver's view about how much utility each item can extract.

$$N \neq P^T \quad \text{unless} \quad U_S = U_R$$

Item Distance:
$$d_i(v_p, v_q) = \| N(\cdot p) - N(\cdot q) \|$$

User Distance:
$$d_u(u_p, u_q) = \| P(\cdot p) - P(\cdot q) \| .$$

Singular Value Decomposition of N

N = Netflix Matrix.

$\tilde{N} = UDV^T \qquad \tilde{N}$ = rank-k.

U and V are orthonormal $\langle u_i, u_j \rangle = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{o.w.} \end{cases}$

$\langle v_i, v_j \rangle = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{o.w.} \end{cases}$

D = Diagonal
with positive entries.

$k \ll n \ll m$

We want $\|\tilde{N} - N\| < \varepsilon_k$.

$$VD^{-1}U^T \times UDV^T = VD^{-1}DV^T = VV^T = I$$
$$UDV^T \times VD^{-1}U^T = UDD^{-1}U^T = UU^T = I.$$
$$\Rightarrow \quad VD^{-1}U^T = \text{Inverse of } \tilde{N}$$

Columns of U = Left Singular Vectors of $\tilde{N}$
$\in \mathbb{R}^m \Rightarrow$ INDEPENDENT FEATURE SET for U

Columns of V = Right Singular Vectors of $\tilde{N}$
$\in \mathbb{R}^n \Rightarrow$ INDEPENDENT FEATURE SET for V.