Q1. [10] *A Repeated Game Against Nature*: You are playing a game against nature. You start with no money. Nature throws a 6-sided dice repeatedly. After each throw, if 1 appears, you win \$1; if 2 appears, you win \$2, etc. But *if 6 appears, you lose all your money and the game ends*. Should you play this game? How should you play? How much can you expect to win (or just describe how you would compute this)?

SOLN.1 *Yes, you should play this game, since you can only win!*

Suppose you have won an amount equal to \$N so far, and you must decide whether to stop or to continue. At this point, your expected winning after the next throw is

$$\frac{1}{6}\left(1 + 2 + 3 + 4 + 5 - N\right) = \frac{15 - N}{6}.$$

So as long as you have less than \$15, you should continue to play.

Note that you will stop, when N takes one of the following values: $\{15, 16, 17, 18, 19\}$. You cannot stop with a value of $N < 15$. You will never reach $N \geq 20$, because the only way you can get to \$20 or anything bigger, is by a dice-throw of 5 after you have reached \$15. So to compute the expected winning, we need to calculate the probabilities of reaching a number between 15 and 19: p_{15}, p_{16}, p_{17}, p_{18} and p_{19}, without encountering a dice-throw of 6. One can use a dynamic programming algorithm or Monte-Carlo simulation to compute these probabilities. Then the expected winning is

$$15p_{15} + 16p_{16} + 17p_{17} + 18p_{18} + 19p_{19} = 6.18.$$