LOGIC

HW #2

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5 November 2013 (due in 2 weeks)

Q1. [10] The axioms of PA in $\mathcal{L}_{ar} := \mathcal{L}\{0, S, +, \cdot\}$ are as follows:

$\forall x\ Sx \neq 0$

$\forall x\ x + 0 = x$ $\qquad$ $\forall x\ x \cdot 0 = 0$

$\forall xy\ (Sx = Sy \rightarrow x = y)$

$\forall xy\ x + Sy = S(x + y)$ $\qquad$ $\forall xy\ x \cdot Sy = x \cdot y + x$

$\phi_0^x \wedge \forall x\ (\phi \rightarrow \phi_{Sx}^x) \rightarrow \forall x\ \phi \quad (IS)$

Prove in PA the associativity, commutativity, and distributivity of $+, \cdot$.

Q2. [10] Define $\leq$ in $\mathcal{L}_{ar}$. Derive reflexivity and transitivity of $\leq$ in PA.