

**\*LOGIC\***

**QUIZ #7**

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5 November 2013

Once again, we visit several Islands in the Archipelago of Knights and Knaves along with our Antropologist. In these islands, those called *knights* always tell the truth and *knaves* always lie. Furthermore, each inhabitant is either a knight or a knave.

Notation:  $k$  = knight,  $s$  = smoker.

Q1. [5 ] On one of these islands, all the inhabitants said the same thing: namely, "All of us here are the same type." What can be deduced about the inhabitants of that island?

Soln1. Let the statement  $S$  be  $S \equiv (\forall x x = k) \vee (\forall x x = \neg k)$ . Since all of them asserted  $S$  (which is either true or false), they are all of same type. Hence  $S = \text{true}$  and the island's inhabitants are knights.

Q2. [5 ] On another of these islands, all the inhabitants said the same thing: namely, "Some of us here are knights and some are knaves." What can be deduced about the inhabitants of that island?

Soln2. Let the statement  $S'$  be  $S' \equiv (\exists x x = k) \wedge (\exists x x = \neg k)$ . Since all of them asserted  $S'$  (which is either true or false), they are all of same type (either all knights or all knaves). Hence  $S' = \text{false}$  and the island's inhabitants are knaves.

Q3. [10 ] On yet another of these islands, all the inhabitants said the same thing: namely, "All knights on this island are smokers." What can be deduced about the inhabitants of that island?

Soln3. Let the statement  $Q$  be  $Q \equiv \forall x (x = k) \Rightarrow (x = s)$ . Since all of them asserted  $Q$  (which is either true or false), they are all of same type (either all knights or all knaves). If they are all knaves, then  $Q$  is false, i.e.,  $\exists x x = k \wedge x \neq s$  is true – which is impossible since there is no knight in the island. If they are all knights, then  $Q = \text{true}$ , and they are all knights and they all smoke.