G22.1170: Fundamental Algorithms
Final Take-Home Exam
(Due Tuesday December 19 2000)

Problem. 1 Order the following functions in increasing order of growth:

(a) $n!$	(b) $\lg \lg n$	(c) $n^{\lg \lg n}$	(d) $n^{1/\lg n}$
(e) $2^{3 \lg n}$	(f) n^2	(g) $(\lg \lg n)^n$	(h) $n \lg^2 n$

Problem. 2 The input is a sequence of n elements x_1, x_2, …, x_n that we can read *sequentially*. We want to use a memory that can only store $O(k)$ elements at a time. Give a high level description of an algorithm that finds the kth *smallest element* in $O(n)$ time.

Problem. 3 Let L be a sequence of n elements. If x and y are pointers into list L then Insert(x) inserts a new element immediately to the right of x, Delete(x) deletes the element to which x points and Order(x, y) returns true if x is before y in the list. Show how to implement all three operations with worst case time $O(\log n)$.

Problem. 4 A simple undirected graph $G = (V, E)$ without self-loops has at most one edge between every pair of vertices and no edge from a vertex to itself. A graph is p-colorable if all vertices can be assigned one of p colors with no edge receiving the same color at both of its ends.

Let $d(v)$ denote the degrees of a vertex v, i.e., the number of edges incident at v. let $d(G)$ denote $\max_{v \in V} d(v)$, the maximum degree of the vertices of the graph G.

Design an efficient algorithm and prove its correctness, which determines $(d(G) + 1)$-coloring of the graph.