

Review of *How Not To Be Wrong: The Power of Mathematical Thinking*, by Jordan Ellenberg (Penguin Press, 2014, 468 pps.)

In *How Not To Be Wrong*, Jordan Ellenberg takes on the daunting task of explaining to a lay audience why they should care about mathematics. In many ways, he succeeds brilliantly. The book is entertaining, informative, wise, and extremely well-written.

Ellenberg's specific objective is to explore the interaction of mathematical reasoning and common sense and to show how math is a powerful extension of common sense. "Math is like an atomic-powered prosthesis that you attach to your common sense, vastly multiplying its reach and strength," he writes. "The problems that we think about every day — problems of politics, of medicine, of commerce, of theology — are shot through with mathematics."

The most visible interactions of mathematics and common sense in recent years, and the most common applications of mathematics to problems of politics, commerce, and medicine, have been in probability and statistics; and accordingly, about two-thirds of Ellenberg's book deals with the basic issues in those areas. He also deals with the dangers of mindless linear extrapolation; the dangers of reporting a percentage of a sum formed of both positive and negative terms; the difficulties of finding a good voting scheme; and formalism in mathematical philosophy. A wide range of mathematical topics, including projective geometry, finite geometry, Lobachevskian geometry, error-correcting codes, the distribution of primes, the theory of the reals, and the non-standard theory of the reals, make cameo appearances.

Ellenberg succeeds in making his arguments without requiring any mathematical background beyond arithmetic and very basic geometry. The many diagrams are almost all crudely hand-drawn, a very wise decision. The book includes only one non-trivial proof; but that one — Barbier's solution of Buffon's needle problem — is a beaut.

At his best, Ellenberg is about as good a science writer as any I've come across. He achieves his best in his first chapter with a story about Abraham Wald. During World War II, Wald had the task of recommending to the Air Force where additional armor should be added to fighter planes. The key data was the relative frequency of bullet holes in different parts of the planes, after they returned from their missions; on average, there were 1.11 bullet holes per square foot in the engine; 1.73 in the fuselage; 1.55 in the fuel system 1.85 in the rest of the plane. Wald realized that the reason for the difference was not that the engine was being hit less often, but that planes hit in the engine were less likely to come back; therefore, the engine was the most important place to add armor.

In terms of pure writing technique, Ellenberg's pacing here is particularly admirable; he pulls off the trick of moving the story along, while making it seem as if he had all the time in the world. The writing is equally impressive — both crystal clear and captivating — in many other sections as well.

I wrote above that the book is wise. What I mean is that it has many of the intellectual virtues that I most value. Ellenberg is strikingly fair-minded on very contentious topics; for instance, his account of the debate between frequentist and Bayesian statistics is as balanced as I have seen. (I think he could say more about the difficulties, in the Bayesian approach, of choosing a hypothesis space and assigning priors; but that's just a hobbyhorse of my own.) He has a very strong sense of history, and a deep knowledge of it. He deals fairly with historical figures, making a serious effort to understand why they took the approaches and reached the conclusions they did, in a manner reminiscent of Stephen Jay Gould's essays. He is very aware of the limitations of the mathematical approach, the meaninglessness of overly precise numbers, the meaninglessness of assigning a number when all that exists is a partial ordering. He decries the cult of the genius in mathematical mythology. He has very good taste about when to put himself into the story.

The book does have its flaws. The most conspicuous is that Ellenberg has a tendency to go on

too long, sometimes much too long. He would have benefitted from a more hard-hearted editor. He has almost forty pages on an interminable story of how some MIT students and some other groups succeeded in regularly making money from a Massachusetts lottery, and another twenty pages on how people to find predictions from letter patterns in the Hebrew Bible; two pages on each would have been more than enough. It is not safe, though, just to skip these two chapters; the former contains, as a digression, the beautiful proof of Buffon's needle, and the latter contains the important discussion of the Baltimore stockbroker.

In a similar way, the book sometimes goes beyond its mandate, in ways that I find unhelpful. I do not think that there is anything to be learned from comparing Antonin Scalia's formalist view of law with Hilbert's formalist view of mathematics; or, at least, learning anything from the comparison would require a much deeper analysis than is possible in a book like this one. In fact, I would have cut the entire chapter on mathematical foundations; it is well-worn ground, and it does not contribute to the question of how not to be wrong.

A deeper problem is that the book does not exactly do what it sets out to. As I wrote above, the ostensible aim is to show how math extends common sense. However, most of the examples in the book are not about that; they are, rather, about cases where there are multiple mathematical arguments; often, these are adjudicated using common sense rather than using more mathematics. To a large extent, therefore, the subject is as much, "How invalid use of mathematics can confuse you," as "How correct use of mathematics can empower you,". (In some respects, the book is an updating of Darrell Huff's sixty-year old classic *How to Lie with Statistics*.) Of course, this itself is an enormously important subject; but the discrepancy between the stated aim and the actual contents leaves the reader a little confused as to what has been accomplished.

Many of the debates here necessarily end up unresolved. Ellenberg's last chapter is a defense of the wisdom of being in doubt. He contrasts a quote of Teddy Roosevelt:

It is not the critic who counts; not the man who points out how the strong man stumbles, or where the doer of deeds could have done them better. The credit belongs to the man who is actually in the arena . . . who errs, who comes short again and again because there is no effort without error and shortcoming.

with a quote from John Ashberry, "For this is action, this not being sure", and he sides with Ashberry. I have mixed feelings. Skepticism and intellectual caution are valuable qualities, particularly in these days of ubiquitous hype, but it is important to recognize their limitations. It is, after all, very easy not to be wrong by the expedient of being too wise to commit to any answer. If one relies on this too much, one can end up like Gattling in Stephen Potter's *Lifemanship*:

[Gattling] was one of the most ignorant and ill-educated men I have ever met, and it was therefore always a particular pleasure to hear him say, to a perfectly ordinary question, "I don't know" slowly, kindly, and distinctly. He was able to indicate, by the tone of his voice, that although he know practically everything about practically everything, and almost everything about this really, yet the mere fact that he knew such a tremendous lot about it made him realise, as we couldn't possibly, that the question was so inextricably two-sided that only a smart-Alec would ever dream of trying to pass judgement either way.

These complaints aside, though, the book is a splendid accomplishment, and very well worth reading, whatever your level of expertise. I learned a lot, and got a clearer view in my mind, about all kinds of things — writing and teaching technique, history and biographical anecdotes, fallacies, social science, even a little math — and had a very enjoyable time learning it.