1. Read Chapter 4 if you haven’t already

2. Start reading Chapter 6 (we will skip over 5 for now and return to it).

3. Prove that the difference of two odd integers is even.

4. Prove that the sum of an even integer and an odd integer is odd.

5. If the following are false, show a counterexample. If they are true, argue that they are (no need for a formal proof) for all real numbers:
   a. \( \lfloor x - y \rfloor = x - \lfloor y \rfloor \)
   b. \( \lfloor x - 1 \rfloor = \lfloor x \rfloor - 1 \)
   c. \( \lceil x + y \rceil = \lceil x \rceil + \lfloor y \rfloor \)
   d. \( \lceil x + 1 \rceil = \lfloor x \rfloor + 1 \)

6. Prove that \( \sqrt{3} \) is irrational by contradiction approach.

7. Prove that for any integer \( n \), \( n^2 \) is odd \( \iff n \) is odd.

8. Prove the average of two odd numbers is even.

9. Section 4.2, Problems 1, 2, 3, 4, 25

10. Section 4.3, Problems 1, 2, 3

11. Prove via contradiction that for any integer \( a \geq 2 \) and any integer \( k \geq 2 \), if \( k \) divides \( a \), then \( k \) does not divide \( (a + 1) \).

12. If the following statements are true, prove them. If not, disprove via counterexample.
   a. If \( a \) divides \( b \) and \( a \) divides \( c \), then \( a \) divides \( b + c \).
   b. If \( a \) divides \( b + c \), then \( a \) divides \( b \) and \( a \) divides \( c \).

13. The following definitions follow from class:

   For integers \( a \) and \( b \) (not equal to zero), the largest integer \( d \) that divides both \( a \) and \( b \) is called
the greatest common divisor of \( a \) and \( b \), and we write this as \( \gcd(a, b) = d \).

For example, \( \gcd(24, 36) = 12 \).

For integers \( a \) and \( b \) (not equal to zero), the smallest positive integer \( c \) that is divisible by both \( a \) and \( b \) is called the least common multiple of \( a \) and \( b \), and we write this as \( \text{lcm}(a, b) = c \). For example, \( \text{lcm}(24, 36) = 72 \).

Once you get a feeling for the above definitions (try some problems from the book), prove or disprove the following statement:

For positive integers \( a \) and \( b \), \( a \cdot b = \gcd(a, b) \cdot \text{lcm}(a, b) \).

**Hint:** Consider the prime factorizations of \( a \) and \( b \). Namely, \( a = p_1^{a_1} p_2^{a_2} \cdots p_n^{a_n} \) and \( b = p_1^{b_1} p_2^{b_2} \cdots p_n^{b_n} \). Think about how the prime factorizations of \( a \) and \( b \) relate to \( \gcd(a, b) \) and \( \text{lcm}(a, b) \).

14. Prove or disprove the following:

a. For all real numbers \( x \), \( \left\lfloor \frac{x}{2} \right\rfloor = \left\lfloor \frac{x}{4} \right\rfloor \).

b. For all positive integers \( n \) and \( k \), \( \left\lfloor \frac{n}{k} \right\rfloor = \left\lfloor \frac{(n-1)}{k} \right\rfloor + 1 \).