Problem 1 (CLRS 22.2-2). (1 point) Show the $d$ and $\pi$ values that result from running breadth-first search on the undirected graph of Figure 22.3, using vertex $u$ as the source.

Problem 2 (CLRS 22.2-8). (3 points) The diameter of a tree $T = (V, E)$ is defined as $\max_{u, v \in V} \delta(u, v)$, that is, the largest of all shortest-path distances in the tree. Give an efficient algorithm to compute the diameter of a tree, prove the algorithm correct, analyze the running time.

Problem 3 (CLRS 22.3-2). (1 point) Show how depth-first search works on the graph of Figure 22.6. Assume that the for loop of lines 5-7 of the DFS procedure considers the vertices in alphabetical order, and assume that each adjacency list is ordered alphabetically. Show the discovery and finishing times for each vertex, and show the classification of each edge.

Problem 4 (CLRS 22.4-1). (1 point) Show the ordering of vertices produced by Topological-Sort when it is run on the dag of Figure 22.8, under the assumption of Exercise 22.3-2.

Problem 5 (CLRS 22.4-3). (2 points) Give an algorithm that determines whether or not a given undirected graph $G = (V, E)$ contains a cycle. Your algorithm should run in $O(V)$ time, independent of $|E|$.

Problem 6 (CLRS 22.5-2). (1 point) Show how the procedure Strongly-Connected-Components works on the graph of Figure 22.6. Specifically, show the finishing times computed in line 1 and the forest produced in line 3. Assume that the loop of lines 5-7 of DFS considers vertices in alphabetical order and that the adjacency lists are in alphabetical order.

Problem 7 (CLRS 22-1). (2 points) A depth-first forest classifies the edges of a graph into tree, back, forward, and cross edges. A breadth-first tree can also be used to classify the edges reachable from the source of the search into the same four categories.

(a) Prove that in a breadth-first search of an undirected graph, the following properties hold:

1. There are no back edges and no forward edges.
2. For each tree edge $(u, v)$ we have $v.d = u.d + 1$.
3. For each cross edge $(u, v)$, we have $v.d = u.d$ or $v.d = u.d + 1$.

(b) Prove that in a breadth-first search of a directed graph, the following properties hold:

1. There are no forward edges.
2. For each tree edge $(u, v)$, we have $v.d = u.d + 1$.
3. For each cross edge $(u, v)$, we have $v.d \leq u.d + 1$.
4. For each back edge $(u, v)$, we have $0 \leq v.d \leq u.d$.