Fundamental Algorithms
CSCI-GA.1170-001/Summer 2017

Homework 4

Problem 1. (1 point) Illustrate the operation of randomized quicksort on the array:
A = (19, 2, 11, 14, 7, 17, 4, 3, 5, 15)
By showing the values in array A after each call to partition.

Problem 2 (CLRS 7.2-5). (2 points) Suppose that the splits at every level of quicksort are in
the proportion $1 - \alpha$ to $\alpha$, where $0 < \alpha \leq 1/2$ is a constant. Show that the minimum depth of a
leaf in the recursion tree is approximately $-\lg n/\lg \alpha$ and the maximum depth is approximately
$-\lg n/\lg(1 - \alpha)$. (Don't worry about integer round-off.)

Problem 3 (CLRS 7.2-6). (3 points) Argue that for any constant $0 < \alpha \leq 1/2$, the probability is
approximately $1 - 2\alpha$ that on a random input array, partition produces a split more balanced
than $1 - \alpha$ to $\alpha$.

Problem 4 (CLRS 7.4-3). (2 points) Show that the expression $q^2 + (n - q - 1)^2$ achieves a
maximum over $q = 0, 1, ..., n - 1$ when $q = 0$ or $q = n - 1$.

Problem 5 (CLRS 7.4-2). (3 points) Show that quicksort's best-case running time is $\Omega(n \lg n)$. 