Administration

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    http://cs.nyu.edu/courses/summer16/CSCI-GA.2340-001/

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• TA/Office Hours, etc
Logic of Statements

• Logical Form and Logical Equivalence
• Conditional Statements
• Valid and Invalid Arguments
• Digital Logic Circuits
• Number Systems & Circuits for Addition
Logical Form

• Initial terms in logic: sentence, true, false
• Statement (proposition) is a sentence that is true or false but not both
• Compound statement is a statement built out of simple statements using logical operations: negation, conjunction, disjunction
Logical Form

• Truth table
• Precedence of logical operations
• English words to logic:
  – It is not hot but it is sunny
  – It is neither hot nor sunny
• Statement form (propositional form) is an expression made up of statement variables and logical connectives (operators)
• Exclusive OR: XOR
Logical Form

• Truth table for \((\neg p \land q) \lor (q \land \neg r)\)
• Two statements are called logically equivalent if and only if (iff) they have identical truth tables
• Double negation
• Non-equivalence: \(\neg(p \lor q)\) vs \(\neg p \lor \neg q\)
• De Morgan’s Laws:
  – The negation of and AND statement is logically equivalent to the OR statement in which component is negated
  – The negation of an OR statement is logically equivalent to the AND statement in which each component is negated
Logical Form

- Applying De-Morgan’s Laws:
  - Write negation for
    - The bus was late or Tom’s watch was slow
    - \(-1 < x \leq 4\)

- Tautology is a statement that is always true regardless of the truth values of the individual logical variables

- Contradiction is a statement that is always false regardless of the truth values of the individual logical variables
Logical Equivalence

- Commutative laws: $p \land q = q \land p$, $p \lor q = q \lor p$
- Associative laws: $(p \land q) \land r = p \land (q \land r)$, $(p \lor q) \lor r = p \lor (q \lor r)$
- Distributive laws: $p \land (q \lor r) = (p \land q) \lor (p \land r)$
  $p \lor (q \land r) = (p \lor q) \land (p \lor r)$
- Identity laws: $p \land t = p$, $p \lor c = p$
- Negation laws: $p \lor \neg p = t$, $p \land \neg p = c$
- Double negative law: $\neg(\neg p) = p$
- Idempotent laws: $p \land p = p$, $p \lor p = p$
- De Morgan’s laws: $\neg(p \land q) = \neg p \lor \neg q$, $\neg(p \lor q) = \neg p \land \neg q$
- Universal bound laws: $p \lor t = t$, $p \land c = c$
- Absorption laws: $p \lor (p \land q) = p$, $p \land (p \lor q) = p$
- Negation of $t$ and $c$: $\neg t = c$, $\neg c = t$
Conditional Statements

• If something, then something: \( p \rightarrow q \), \( p \) is called the hypothesis and \( q \) is called the conclusion.

• The only combination of circumstances in which a conditional sentence is false is when the hypothesis is true and the conclusion is false.

• A conditional statement is called vacuously true or true by default when its hypothesis is false.

• Among \( \wedge, \lor, \neg \) and \( \rightarrow \) operations, \( \rightarrow \) has the lowest priority.
Conditional Statements

• Write truth table for: \( p \land q \rightarrow \neg p \)
• Show that \((p \lor q) \rightarrow r = (p \rightarrow r) \land (q \rightarrow r)\)
• Representation of \(\rightarrow\): \(p \rightarrow q = \neg p \lor q\)
• Re-write using if-else: Either you get in class on time, or you risk missing some material
• Negation of \(\rightarrow\): \(\neg(p \rightarrow q) = p \land \neg q\)
• Write negation for: If it is raining, then I cannot go to the beach
Conditional Statements

• Contrapositive $p \rightarrow q$ is another conditional statement $\sim q \rightarrow \sim p$
• A conditional statement is equivalent to its contrapositive
• The converse of $p \rightarrow q$ is $q \rightarrow p$
• The inverse of $p \rightarrow q$ is $\sim p \rightarrow \sim q$
• Conditional statement and its converse are not equivalent
• Conditional statement and its inverse are not equivalent
Conditional Statements

- The converse and the inverse of a conditional statement are equivalent to each other
- \( p \) only if \( q \) means \( \sim q \rightarrow \sim p \), or \( p \rightarrow q \)
- Biconditional of \( p \) and \( q \) means “\( p \) if and only if \( q \)” and is denoted as \( p \leftrightarrow q \)
- \( r \) is a sufficient condition for \( s \) means “if \( r \) then \( s \)”
- \( r \) is a necessary condition for \( s \) means “if not \( r \) then not \( s \)”
Exercises

• Write contrapositive, converse and inverse statements for:
  – If $P$ is a square, then $P$ is a rectangle
  – If today is Thanksgiving, then tomorrow is Friday
  – If $c$ is rational, then the decimal expansion of $r$ is repeating
  – If $n$ is prime, then $n$ is odd or $n$ is 2
  – If $x$ is nonnegative, then $x$ is positive or $x$ is 0
  – If Tom is Ann’s father, then Jim is her uncle and Sue is her aunt
  – If $n$ is divisible by 6, then $n$ is divisible by 2 and $n$ is divisible by 3
Arguments

• An argument is a sequence of statements. All statements except the final one are called premises (or assumptions or hypotheses). The final statement is called the conclusion.

• An argument is considered valid if from the truth of all premises, the conclusion must also be true.

• The conclusion is said to be inferred or deduced from the truth of the premises.
Arguments

• Test to determine the validity of the argument:
  – Identify the premises and conclusion of the argument
  – Construct the truth table for all premises and the conclusion
  – Find critical rows in which all the premises are true
  – If the conclusion is true in all critical rows then the argument is valid, otherwise it is invalid

• Example of valid argument form:
  – Premises: p ∨ (q ∨ r) and ~r, conclusion: p ∨ q

• Example of invalid argument form:
  – Premises: p → q ∨ ~r and q → p ∧ r, conclusion: p → r
Valid Argument-Forms

• Modus ponens (method of affirming):
  – Premises: $p \rightarrow q$ and $p$, conclusion: $q$

• Modus tollens (method of denying):
  – Premises: $p \rightarrow q$ and $\neg q$, conclusion: $\neg p$

• Disjunctive addition:
  – Premises: $p$, conclusion: $p \mid q$
  – Premises: $q$, conclusion: $p \mid q$

• Conjunctive simplification:
  – Premises: $p \& q$, conclusion: $p$, $q$
Valid Argument-Forms

• Disjunctive Syllogism:
  – Premises: p | q and ~q, conclusion: p
  – Premises: p | q and ~p, conclusion: q

• Hypothetical Syllogism
  – Premises: p → q and q → r, conclusion: p → r

• Dilemma: proof by division into cases:
  – Premises: p | q and p → r and q → r, conclusion: r
Complex Deduction

• Premises:
  – If my glasses are on the kitchen table, then I saw them at breakfast
  – I was reading the newspaper in the living room or I was reading
    the newspaper in the kitchen
  – If I was reading the newspaper in the living room, then my glasses
    are on the coffee table
  – I did not see my glasses at breakfast
  – If I was reading my book in bed, then my glasses are on the bed
    table
  – If I was reading the newspaper in the kitchen, then my glasses are
    on the kitchen table

• Where are the glasses?
Fallacies

• A fallacy is an error in reasoning that results in an invalid argument

• Three common fallacies:
  – Vague or ambiguous premises
  – Begging the question (assuming what is to be proved)
  – Jumping to conclusions without adequate grounds

• Converse Error:
  – Premises: p → q and q, conclusion: p

• Inverse Error:
  – Premises: p → q and ~p, conclusion: ~q
Fallacies

• It is possible for a valid argument to have false conclusion and for an invalid argument to have a true conclusion:
  – Premises: if John Lennon was a rock star, then John Lennon had red hair, John Lennon was a rock star; Conclusion: John Lennon had red hair
  – Premises: If New York is a big city, then New York has tall buildings, New York has tall buildings; Conclusion: New York is a big city
Contradiction

• Contradiction rule: if one can show that the supposition that a statement p is false leads to a contradiction, then p is true.

• Knight is a person who always says truth, knave is a person who always lies:
  – A says: B is a knight
  – B says: A and I are of opposite types

What are A and B?
Digital Logic Circuits

- Digital Logic Circuit is a basic electronic component of a digital system.
- Values of digital signals are 0 or 1 (bits).
- Black Box is specified by the signal input/output table.
- Three gates: NOT-gate, AND-gate, OR-gate.
- Combinational circuit is a combination of logical gates.
- Combinational circuit always correspond to some boolean expression, such that input/output table of a table and a truth table of the expression are identical.
Number Systems

- Decimal number system
- Binary number system
- Conversion between decimal and binary numbers
- Binary addition and subtraction
Negative Numbers

• Two’s complement of a positive integer $a$ relative to a fixed bit length $n$ is the binary representation of $2^n - a$

• To find an 8-bit complement:
  – Write 8-bit binary representation of the number
  – Flip all bits (one’s complement)
  – Add 1 to the obtained binary

• Addition of negative numbers
Hexadecimal Numbers

• Hexadecimal notation is a number system with base 16
• Digits of hexadecimal number system
• Conversion between hexadecimal and binary and hexadecimal and decimal systems