Problem Set 6

Assigned: July 1
Due: July 8

Problem 1

Trace the execution of Dijkstra’s algorithm on the following graph, taking A as the starting vertex. Show the successive states of the array $D[i]$ and of the set of vertices whose distance has been determined.

Problem 2 (Siegel Ex. 8.2)

Write an enhancement to the Floyd-Warshall algorithm that saves, in $\text{Intermediate}[i,j]$, the last vertex before $j$ on the shortest path from $i$ to $j$. Notice that proper initialization makes the algorithm easier.

Problem 3

Consider the following problem. You are given a directed graph $G$ with two disjoint subsets $P$ and $Q$. A path is considered invalid if it goes first through a vertex in $P$ and then through a vertex in $Q$. For example $P$ and $Q$ may be points in enemy countries, and $Q$ may prohibit travellers whose passport has a visa to $P$. Or in an epidemic of a communicable disease, one may want to block people who have been through $Q$ from entering $B$.

For example, in the graph in problem 1, if $U = \{B,C\}$ and $V = \{D,E\}$ then the path $A \rightarrow B \rightarrow F \rightarrow E \rightarrow H \rightarrow I$ is invalid, because it first goes through $U$ at vertex $B$ and then later goes through $V$, at vertex $E$. The path $A \rightarrow D \rightarrow B \rightarrow F \rightarrow I$ is valid, because $D$ in $V$ comes before $B$ in $U$.

Describe how the method of cloning can be used to find the optimal valid path.