Discrete Mathematics
Lecture 1
Logic of Compound Statements

Harper Langston
New York University
Administration

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  http://cs.nyu.edu/courses/summer14/CSCI-GA.2340-001/

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Logic of Statements

• Logical Form and Logical Equivalence
• Conditional Statements
• Valid and Invalid Arguments
• Digital Logic Circuits
• Number Systems & Circuits for Addition
Logical Form

• Initial terms in logic: sentence, true, false
• Statement (proposition) is a sentence that is true or false but not both
• Compound statement is a statement built out of simple statements using logical operations: negation, conjunction, disjunction
Logical Form

• Truth table
• Precedence of logical operations
• English words to logic:
  – It is not hot but it is sunny
  – It is neither hot nor sunny
• Statement form (propositional form) is an expression made up of statement variables and logical connectives (operators)
• Exclusive OR: XOR
Logical Form

- Truth table for \((\neg p \land q) \lor (q \land \neg r)\)
- Two statements are called logically equivalent if and only if (iff) they have identical truth tables
- Double negation
- Non-equivalence: \(\neg(p \lor q)\) vs \(\neg p \lor \neg q\)
- De Morgan’s Laws:
  - The negation of an AND statement is logically equivalent to the OR statement in which component is negated
  - The negation of an OR statement is logically equivalent to the AND statement in which each component is negated
Logical Form

• Applying De-Morgan’s Laws:
  – Write negation for
    • The bus was late or Tom’s watch was slow
    • -1 < x <= 4

• Tautology is a statement that is always true regardless of the truth values of the individual logical variables

• Contradiction is a statement that is always false regardless of the truth values of the individual logical variables
Logical Equivalence

- Commutative laws: \( p \land q = q \land p, \ p \lor q = q \lor p \)
- Associative laws: \((p \land q) \land r = p \land (q \land r), \ (p \lor q) \lor r = p \lor (q \lor r)\)
- Distributive laws: \( p \land (q \lor r) = (p \land q) \lor (p \land r) \)
\[
p \lor (q \land r) = (p \lor q) \land (p \lor r)
\]
- Identity laws: \( p \land t = p, \ p \lor c = p \)
- Negation laws: \( p \lor \neg p = t, \ p \land \neg p = c \)
- Double negative law: \( \neg(\neg p) = p \)
- Idempotent laws: \( p \land p = p, \ p \lor p = p \)
- De Morgan’s laws: \( \neg(p \land q) = \neg p \lor \neg q, \ \neg(p \lor q) = \neg p \land \neg q \)
- Universal bound laws: \( p \lor t = t, \ p \land c = c \)
- Absorption laws: \( p \lor (p \land q) = p, \ p \land (p \lor q) = p \)
- Negation of \( t \) and \( c \): \( \neg t = c, \ \neg c = t \)
Conditional Statements

- If something, then something: $p \rightarrow q$, $p$ is called the hypothesis and $q$ is called the conclusion
- The only combination of circumstances in which a conditional sentence is false is when the hypothesis is true and the conclusion is false
- A conditional statements is called vacuously true or true by default when its hypothesis is false
- Among $\land$, $\lor$, $\sim$ and $\rightarrow$ operations, $\rightarrow$ has the lowest priority
Conditional Statements

- Write truth table for: \( p \land q \rightarrow \neg p \)
- Show that \((p \lor q) \rightarrow r = (p \rightarrow r) \land (q \rightarrow r)\)
- Representation of \( \rightarrow \): \( p \rightarrow q = \neg p \lor q \)
- Re-write using if-else: Either you get in class on time, or you risk missing some material
- Negation of \( \rightarrow \): \( \neg(p \rightarrow q) = p \land \neg q \)
- Write negation for: If it is raining, then I cannot go to the beach
Conditional Statements

- Contrapositive $p \rightarrow q$ is another conditional statement $\sim q \rightarrow \sim p$
- A conditional statement is equivalent to its contrapositive
- The converse of $p \rightarrow q$ is $q \rightarrow p$
- The inverse of $p \rightarrow q$ is $\sim p \rightarrow \sim q$
- Conditional statement and its converse are not equivalent
- Conditional statement and its inverse are not equivalent
Conditional Statements

• The converse and the inverse of a conditional statement are equivalent to each other
• $p$ only if $q$ means $\neg q \rightarrow \neg p$, or $p \rightarrow q$
• Biconditional of $p$ and $q$ means “$p$ if and only if $q$” and is denoted as $p \leftrightarrow q$
• $r$ is a sufficient condition for $s$ means “if $r$ then $s$”
• $r$ is a necessary condition for $s$ means “if not $r$ then not $s$”
Exercises

• Write contrapositive, converse and inverse statements for:
  – If P is a square, then P is a rectangle
  – If today is Thanksgiving, then tomorrow is Friday
  – If c is rational, then the decimal expansion of r is repeating
  – If n is prime, then n is odd or n is 2
  – If x is nonnegative, then x is positive or x is 0
  – If Tom is Ann’s father, then Jim is her uncle and Sue is her aunt
  – If n is divisible by 6, then n is divisible by 2 and n is divisible by 3
Arguments

• An argument is a sequence of statements. All statements except the final one are called premises (or assumptions or hypotheses). The final statement is called the conclusion.

• An argument is considered valid if from the truth of all premises, the conclusion must also be true.

• The conclusion is said to be inferred or deduced from the truth of the premises
Arguments

• Test to determine the validity of the argument:
  – Identify the premises and conclusion of the argument
  – Construct the truth table for all premises and the conclusion
  – Find critical rows in which all the premises are true
  – If the conclusion is true in all critical rows then the argument is valid, otherwise it is invalid

• Example of valid argument form:
  – Premises: $p \lor (q \lor r)$ and $\neg r$, conclusion: $p \lor q$

• Example of invalid argument form:
  – Premises: $p \rightarrow q \lor \neg r$ and $q \rightarrow p \land r$, conclusion: $p \rightarrow r$
Valid Argument-Forms

- Modus ponens (method of affirming):
  - Premises: $p \rightarrow q$ and $p$, conclusion: $q$
- Modus tollens (method of denying):
  - Premises: $p \rightarrow q$ and $\neg q$, conclusion: $\neg p$
- Disjunctive addition:
  - Premises: $p$, conclusion: $p \mid q$
  - Premises: $q$, conclusion: $p \mid q$
- Conjunctive simplification:
  - Premises: $p \& q$, conclusion: $p$, $q$
Valid Argument-Forms

• Disjunctive Syllogism:
  – Premises: \( p \lor q \) and \( \neg q \), conclusion: \( p \)
  – Premises: \( p \lor q \) and \( \neg p \), conclusion: \( q \)

• Hypothetical Syllogism
  – Premises: \( p \to q \) and \( q \to r \), conclusion: \( p \to r \)

• Dilemma: proof by division into cases:
  – Premises: \( p \lor q \) and \( p \to r \) and \( q \to r \),
    conclusion: \( r \)
Complex Deduction

- Premises:
  - If my glasses are on the kitchen table, then I saw them at breakfast
  - I was reading the newspaper in the living room or I was reading the newspaper in the kitchen
  - If I was reading the newspaper in the living room, then my glasses are on the coffee table
  - I did not see my glasses at breakfast
  - If I was reading my book in bed, then my glasses are on the bed table
  - If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table

- Where are the glasses?
Fallacies

• A fallacy is an error in reasoning that results in an invalid argument

• Three common fallacies:
  – Vague or ambiguous premises
  – Begging the question (assuming what is to be proved)
  – Jumping to conclusions without adequate grounds

• Converse Error:
  – Premises: \( p \rightarrow q \) and \( q \), conclusion: \( p \)

• Inverse Error:
  – Premises: \( p \rightarrow q \) and \( \sim p \), conclusion: \( \sim q \)
Fallacies

• It is possible for a valid argument to have false conclusion and for an invalid argument to have a true conclusion:
  – Premises: if John Lennon was a rock star, then John Lennon had red hair, John Lennon was a rock star; Conclusion: John Lennon had red hair
  – Premises: If New York is a big city, then New York has tall buildings, New York has tall buildings; Conclusion: New York is a big city
Contradiction

• Contradiction rule: if one can show that the supposition that a statement p is false leads to a contradiction, then p is true.

• Knight is a person who always says truth, knave is a person who always lies:
  – A says: B is a knight
  – B says: A and I are of opposite types

What are A and B?
Digital Logic Circuits

- Digital Logic Circuit is a basic electronic component of a digital system
- Values of digital signals are 0 or 1 (bits)
- Black Box is specified by the signal input/output table
- Three gates: NOT-gate, AND-gate, OR-gate
- Combinational circuit is a combination of logical gates
- Combinational circuit always correspond to some boolean expression, such that input/output table of a table and a truth table of the expression are identical
Number Systems

• Decimal number system
• Binary number system
• Conversion between decimal and binary numbers
• Binary addition and subtraction
Negative Numbers

- Two’s complement of a positive integer \( a \) relative to a fixed bit length \( n \) is the binary representation of \( 2^n - a \)

- To find an 8-bit complement:
  - Write 8-bit binary representation of the number
  - Flip all bits (one’s complement)
  - Add 1 to the obtained binary

- Addition of negative numbers
Hexadecimal Numbers

• Hexadecimal notation is a number system with base 16
• Digits of hexadecimal number system
• Conversion between hexadecimal and binary and hexadecimal and decimal systems