Homework 1 - Discrete Math - Due 06/11/14

Assigned: 05/29/14
Due: 06/11/14

Please make sure to clearly write your name at the top of your hand-in. Also, indicate if you worked with anybody and also indicate how many hours total you worked on the homework. This looks like more homework than it is since many problems are quite simple and others have solutions in the back. Feel free to discuss any problems (including the bonuses) on the class mailing list. I am also required to remind all students of the academic integrity policy at `http://www.cs.nyu.edu/web/Academic/Graduate/academic_integrity.html`. Any violations of this policy may result in failure of the course and being reported to the head of the department.

Problem 0 Sign up for the mailing list, acquire a copy of the textbook and read chapters 1, 2 (you can skim chapter 1).

Problem 1

Let p,q *and* r be the propositions:
p: You have the flu
q: You miss the final exam
r: You pass the course
Express each of the following as a sentence in English: a) $p \to q$

b) $q \to q$
c) $\neg q \leftrightarrow r$
d) $p \vee q \vee r$
e) $(p \to \neg r) \wedge (q \to \neg r)$
f) $(p \wedge q) \vee (\neg q \wedge r)$

Problem 2

Let p,q *and* r be the propositions:
p: You get an A on the final exam
q: You do every exercise in the book
r: You get an A in the class
Write each of the following using p, q and r:

a) You get an A in the class, but you do not do every exercise in the book
b) You get an A on the final, you do every exercise in the book, and you get an A in the class
c) To get an A in the class, it is necessary for you to get an A on the final

d) You get an A on the final, but you don't do every exercise in the book. Nevertheless, you get an A in the class

e) Getting an A on the final exam and doing every exercise in the book is not sufficient for getting an A in the class

f) You will get an A in the class if and only if you either do every exercise in this book or you get an A on the final

Problem 3 State the converse, contrapositive and inverse of the following sentence: *If it snows tonight, then I will stay home.* Explain why the contrapositive is equivalent to the original statement. Also, explain why the converse and inverse are not equivalent to the original statement, but why they are equivalent to each other.

Problem 4

Construct a truth table for the following statements:

a) $(p \wedge q) \vee \neg r$
b) $(p \rightarrow q) \vee (\neg p \rightarrow r)$

Problem 5

Construct truth tables to verify the following associative, distributive laws, and De Morgan's laws:

a) $(p \vee q) \vee r \equiv p \vee (q \vee r)$
b) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
c) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

Problem 6 Use a truth table to verify the following implication is a tautology:

$[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

Problem 7 For each of the following sets of premises, what relevant conclusion(s) can be reached? Explain which rules of inference are used.

a) "If I play hockey, then I am sore the next day", "I use the whirlpool if I am sore", "I did not use the whirlpool"

b) "I am dreaming or hallucinating", "I am not dreaming", "If I am hallucinating, I see elephants smoking"

Problem 8 Five friends enjoy IMing with each other, and you want to determine who is currently IMing, given the following information. Either K or H , or both are IMing. Either R or V, but not both are IMing. If A is IMing, so is R. V and K are are either both IMing are neither is. If H is IMing, then so are A and K. What can you conclude?

Problem 9 Here is a puzzle by Lewis Carroll. What is the conclusion of the following premises?

(a) No interesting poems are unpopular among people of real taste.
(b) No modern poetry is free from affectation.
(c) All your poems are on the subject of soap-bubbles.
(d) No affected poetry is popular among people of real taste.
(e) No ancient poem is on the subject of soap-bubbles.

Problem 10 Convert the following numbers from decimal to binary notation.

a) 231
b) 4532
c) 10101

Problem 11 Convert the following numbers from binary to decimal notation.

a) 11011
b) 1110111110
c) 1010110101
d) 11111000001111

Problem 12 Integers can be represented as **one's complement**s to simplify computer arithmetic. To present positive and negative integers less than 2^{n-1}, a total of n bits are used: The left-most bit is used to represent the sign. That is, a zero in this position implies a positive integer while a 1 represents a negative integer. For positive integers, the remaining bit positions are identical to a normal binary expansion. For negative integers, we first find the binary expansion of the absolute value of the integer, and then take the complement of each of the bit positions(1 becomes a 0, and 0 becomes a 1), except for the leftmost.

A similar representation is the **two's complement** representation of integers (more commonly used). To represent an integer x where $-2^{n-1} \leq x \leq$

$2^{n-1} - 1$, n bit positions are used. The leftmost bit again represents the sign, where a 0 implies a positive integer, and a 1 represents a negative integer (same as for one's complement). For a positive integer, the remaining bits are the same as for a normal binary expansion. For a negative integer, the remaining bits are the binary expansion of $2^{n-1} - |x|$.

Use the above information to answer the following questions:

a) Fine the one's complement and two's complement of the following integers: 22, 31, -7, -19

b)What integer do the following complement representations of length five represent? Answer for both one's and two's complement expansions:

11001, 01101, 10001, 1111

c) How is the one's complement representation of the sum of two integers obtained from the one's complement representations (that is, given a one's complement representation for integers x_1 and x_2, how do we obtain the one's complement representation for $x_1 + x_2$ without converting x_1 or x_2 to integers)? Also, answer this question for two's complement.

d) Do some research on the Internet and figure out where and when two's complements are actually used. You need not write anything up for this (it's for your own edification).

BONUS Problems Try working on these to the best of your ability

I) The following puzzle is attributed to Einstein and is known as the *zebra puzzle*:

Five men from different countries and different jobs live in consecutive houses on the same street. These houses are painted different colors, and the men have different types of pets and different favorite beverages. Determine who owns a zebra and whose favorite beverage is seltzer given the following: The Englishman lives in the red house, the Spaniard owns a dog, the Japanese man is a painter, the Italian drinks tea, the Norwegian lives in the first house on the left, the green house is immediately to the right of the white one, the photographer breeds snails, the diplomat lives in the yellow house, milk is the favorite drink of the person living in the middle house, the owner of the green house drinks coffee, the Norweigan's house is next to the blue one, the violinist drinks O.J., the fox is the pet in a house next to the physician's house, and a horse is the pet in a house next to the diplomat.

II) Here is a problem that is similar to the Knights and Knaves problem You are walking down the road, searching for lost Mayan gold, and you come upon

a fork in the road, so there is a path to the left and one to the right. Each path is guarded by a soldier with a large machete. In unison they say, "Welcome traveler. You have come to a crossroads where you must decide which path to take. If you choose correctly, you will find unimaginable riches, but if you choose incorrectly, you will die instantly by our swords. You may ask one of us one question and one question only to which we will reply either 'yes' or 'no' and then you must make your decision. Further, if you try to turn back and leave, we will kill you anyway."

Knowing that you must go forward, you sit and ponder for a few minutes (there is no time limit). Suddenly, you turn to the guard on the left and ask your question, to which he replies, "Yes", and then you turn and walk down the left path and find the gold. What question did you ask and what is your reasoning? Your argument can be informal. (I mentioned that this is also in the 80s movie Labyrinth, and I will try to get a copy of this movie to show the clip).

III) Do you think the following argument is valid?

"If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil, he would be impotent. If he were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent. Therefore, Superman does not exist."

State which rules of inference you use, and use p, q, r variables to verify your claim.

IV) Here is a puzzle by Lewis Carroll. What is the conclusion of the following premises?

- No interesting poems are unpopular among people of real taste.
- No modern poetry is free from affectation.
- All your poems are on the subject of soap-bubbles.
- No affected poetry is popular among people of real taste.
- No ancient poem is on the subject of soap-bubbles.

V) We will go over this in class, perhaps next time, but I want you all to start thinking in terms of how to tackle such problems.

A cat traps 99 mice in a room and offers them a proposition: They can all be eaten right away or try to save each other by the following. The 99 mice

will be lined up so that they are all facing the same direction in a straight line. (So, mouse #99 can see the heads of the other 98 mice, and mouse #1 can see nobody). The cat then says that he will place either a blue cap or a red cap on the top of each mouse's head. After they are lined up, they will have to yell out either red or blue, starting from the back of the line (#99), moving forward to the front (#1). If the color they announce matches that on their head, they will live; otherwise, they are eaten and the others won't know (the cat eats quietly). Again, they will be able to see all of the hats in front of them but not behind them or their own hat color.

The mice get together and realize it's better if some live rather than all die. At first they think they could all yell the same color (say blue) and save half of themselves, but there is no guarantee that the colors are split evenly (they might all be red or all red but one or two!). They then realize that they are all pretty good at math, and when one mouse yells a color, they will all be able to hear what he yells. In the end, they decide upon a plan that will save most of them (and that the cat is stupid). How many mice can be saved (what is the minimum and maximum) and why?

Hint : Try solving the problem for a simpler number (i.e., try solving it for 3 mice first and then see if you can extrapolate from there).