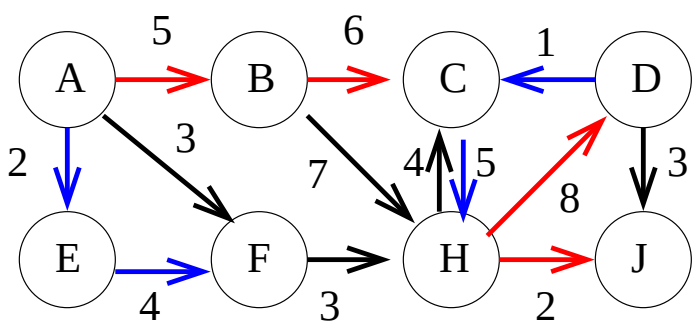
Problem Set 7

Assigned: July 9
Due: July 16

Problem 1.

Suppose that you have a graph G with weighted edges where edges have different colors. Consider the problem of finding the shortest path in which no two consecutive edges have the same color. For instance, in the graph below, if you are looking for the shortest path from A to J, the path A→F→H→J (cost 8) is ruled out because it has two black edges in a row, and the path A→E→F→H→J (cost 11) is ruled out because it has two blue edges in a row. So the shortest path satisfying the constraints is A→B→H→J (cost 14; this has two red edges but they are not consecutive, so that's OK.)



A. Use the method of cloning to construct a new graph G' such that the solution of the ordinary single-source shortest path problem on G' gives the solution to the constrained single-source shortest path problem on G. (Hint: For each vertex v in G other than the source, construct separate copies of v corresponding to the color of the edge taken into v. For example, there would be two copies of vertex H: "H, arriving on a black edge" and "H, arriving on a blue edge".)

B. Let v be the number of vertices in G; let e be the number of edges, and let c be the number of colors. Let v' be the number of vertices in G'. Give arguments to show that $v' \leq c \cdot v$ and $v' \leq e$. Give an upper bound on the number of edges in G'.

Problem 2

As in problem 1, suppose that you have a graph with colored edges, but suppose that we are now ready to accept any path that has at least two different colors. For instance in the graph above, the paths A→B→H and A→E→F→H are OK but the path A→F→H is not, since both the edges are black. (No path consisting of a single edge can be OK.)

A. Use the method of cloning to solve the all-pairs shortest path problem for paths that satisfies this constraint. (Hint: Make $c + 2$ copies of every vertex V: V at the start; V, after following a path with only color C; and V after following a path with at least two colors.).

B. Give bounds on the number of vertices and number of edges in the new graph and the running time of the all-pairs shortest path algorithm as functions of v, e, and c.

Problem 3

A programmer misremembered the Floyd-Warshall algorithm, and programmed it up as follows

```
function FloydWarshall(int[N,N] c) {
    int[N,N] a = copy(c);
    for (i = 1 to n)
      for (j = 1 to n)
         for (k = 1 to n)
            a[i,j] = min(a[i,j], a[i,k] + a[k,j]);
     return a;
    }
```

A. Construct an example of a graph for which this gives the wrong answer.

B. Which of the following three statements is true:

   i. The output of the above algorithm can be smaller than the true answer, but cannot be larger.

   ii. The output of the above algorithm can be larger than the true answer, but cannot be smaller.

   iii. The output of the above algorithm can be either smaller or larger than the true answer.

   Justify your answer.