Problem Set 6

Assigned: July 3
Due: July 10

Problem 1.

Let \( G \) be a DAG where the vertices are labelled with numerical values.

A. Write a function \( \text{MaxReachable}(u) \) which returns the maximum label on a vertex reachable from vertex \( u \) (including \( u \) itself.)

B. Write a function \( \text{TotalReachable}(u) \) which returns the sum of the labels on vertices reachable from vertex \( u \).

C. Write a function \( \text{MaxPathFrom}(u) \) which returns the maximum sum of the labels on any path starting at \( u \).

All these should run in time linear in the size of \( G \).

For example, in the graph below:

\[
\begin{array}{cccc}
A & C & E & G \\
15 & 7 & 8 & 8 \\
B & D & F & H \\
3 & 1 & 10 & -2
\end{array}
\]

\( \text{MaxReachable}(B) = 10 \) (corresponding to F).
\( \text{TotalReachable}(B) = 35 \) (B+C+D+E+F+G+H)
\( \text{MaxPathFrom}(B) = 27 \) (corresponding to B-D-C-E-G).

Problem 2

Suppose that you have two DAGs \( G \) and \( H \), with the same vertices and with different edges. We say that \( G \) and \( H \) are \textit{incompatible} if there is a pair of vertices \( u \neq v \) such that there is a path from \( u \) to \( v \) in \( G \) and a path from \( v \) to \( u \) in \( H \). If there is no such pair, then \( G \) and \( H \) are compatible.

Write a function to test whether two DAGs \( G \) and \( H \) are compatible. This should run in linear time.
Problem 3

Trace the execution of Dijkstra’s algorithm on the following graph, taking A as the starting vertex. Show the successive states of the array $D[i]$ and of the set of vertices whose distance has been determined.