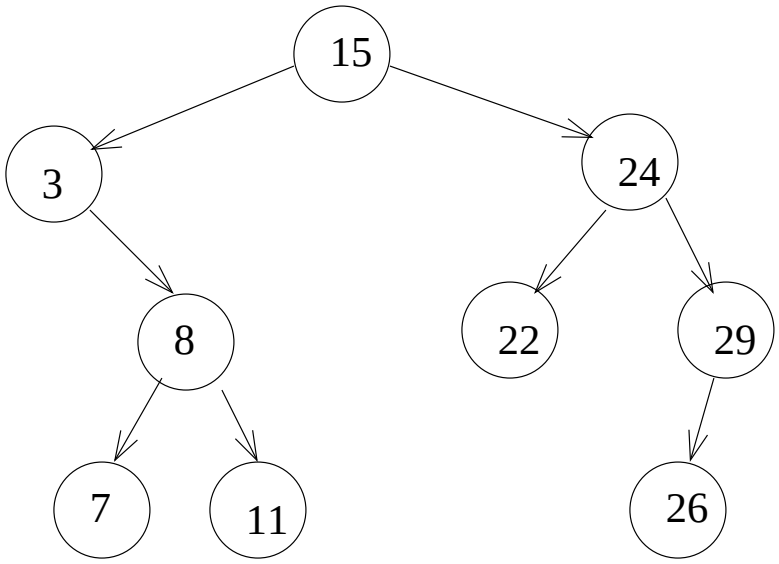
Problem Set 4

Assigned: June 19
Due: June 26

Problem 1

A programmer proposes to implement a binary search tree using an array implementation similar to the array implementation of a heap. As in the implementation of a heap, the children of `a[i]` are at `a[2i+1]` and `a[2i+2]`. For instance, the tree below would be implemented as the array $[15, 3, 24, -, 8, 22, 29, -, -, 7, 11, -, -, 26, -]$. (Another way of viewing this is that this is breadth-first search if you fill in all the missing spaces with null.)



A. Describe the algorithm for searching for an element `x` in this tree.

B. This implementation is rarely if ever used in practice. What is the disadvantage of this method, as compared to constructing a binary search tree from dynamic objects?

C. Some time ago, I gave parts (A) and (B) as problems on an exam. A few of the students came up with the following answer to (B):

> With this implementation, delete can be inefficient in the following case: Suppose that you delete node N with parent P and a single child C by making C a child of P. Then all of the subtree under C will have to be moved in the array.

This answer is actually only half right. (I gave it full credit though, since it was certainly a good enough answer for an exam.)

C.a What is the worst case running time of deleting N if you implement delete as described above?

C.b Find a more efficient method of deleting an internal node with one child with this implementation.

In both C.a and C.b, running time should be given as a function of `n`, the total number of elements, and `h`, the height of the tree.

Problem 2

Modify the definition of a 2-3 tree so that it supports the following operations with the specified running times. You may assume that the reader understands the standard definition of a 2-3 (the one given in class, with all the values in the leaves); all you have to describe are the modifications that need to be made.

Note that we want a *single* (compound) data structure that supports *all* these operations, not different data structures for each operation.

- `add(x)` : Add element `x` to the set. Time: $O(\log(n))$
- `delete(x)` : Delete element `x` from the set. Time: $O(\log(n))$.
- `element?(x)`: Is `x` in the set? Time: $O(\log(n))$.
- `index(i)`: Find the `i`th smallest element in the set. Time: $O(\log(n))$.
- `indexOf(x)`: Find the index of `x` in the set. TIme: $O(\log n)$.
- `subrange(i,k)`: Return `k` elements in the set in sequence starting with the `i`th. For instance, subrange(100,5) should return a list of the 100th, 101st, 102nd, 103rd, and 104th smallest elements. Time: $O(k + \log(n))$.
- `min()`: Find the smallest element in the set. Time: $O(1)$.
- `max()`: Find the largest element in the set. Time: $O(1)$.
- `median()`: Find the median element in the set. For instance if there are 99 or 100 elements in the set, return the 50th. Time: $O(1)$.