Programming Languages

ML

CSCI-GA.2110-001
Summer 2012
- originally developed for use in writing theorem provers
- functional: functions are first-class values
- garbage collection
- strict evaluation (applicative order)
- strong and static typing; powerful type system
  - parametric polymorphism
  - structural equivalence
  - all with type inference!
- advanced module system
- exceptions
- miscellaneous features:
  - datatypes (merge of enumerated literals and variant records)
  - pattern matching
  - `ref` type constructor (like “const pointers” (not “pointers to const”))
val k = 5;  
val k = 5 : int

- k * k * k;
val it = 125 : int

- [1, 2, 3];
val it = [1,2,3] : int list

- ["hello", "world"]; 
val it = ["hello","world"] : string list

- 1 :: [ 2, 3 ];
val it = [1,2,3] : int list

- [ 1, "hello"];
error
- null [1, 2];
val it = false : bool
- null [ ];
val it = true : bool
- hd [1, 2, 3];
val it = 1 : int
- tl [1, 2, 3];
val it = [2, 3] : int list
- [ ];
val it = [ ] : 'a list this list is polymorphic
A function *declaration*:

- `fun abs x = if x >= 0.0 then x else -x`
  
  `val abs = fn : real -> real`

A function *expression*:

- `fn x => if x >= 0.0 then x else -x`
  
  `val it = fn : real -> real`
- fun length xs = 
    if null xs
    then 0
    else 1 + length (tl xs);

val length = fn : 'a list -> int

'a denotes a type variable; length can be applied to lists of any element type

The same function, written in pattern-matching style:

- fun length [] = 0
  | length (x::xs) = 1 + length xs

val length = fn : 'a list -> int
Advantages of type inference and polymorphism:

- frees you from having to write types. A type can be more complex than the expression whose type it is, e.g., `flip`.
- with type inference, you get polymorphism for free:
  - no need to specify that a function is polymorphic
  - no need to ”instantiate” a polymorphic function when it is applied
All functions in ML take exactly one argument.

If a function needs multiple arguments, we can

1. pass a tuple:
   - (53, "hello"); (* a tuple *).
   ```
   val it = (53, "hello") : int * string
   ```
   We can also use tuples to return multiple results.

2. use currying (named after Haskell Curry, a logician).
Another function; takes two lists and returns their concatenation

- fun append1 ([ ], ys) = ys
  | append1 (x::xs, ys) = x :: append1 (xs, ys);
val append1 = fn: 'a list * 'a list -> 'a list

- append1 ([1,2,3], [8,9]);
val it = [1,2,3,8,9] : int list
The same function, written in curried style:

- \[
\text{fun append2} \left[ \right] \rightarrow \text{ys} = \text{ys} \\
| \quad \text{append2} \left( \text{x::xs} \right) \rightarrow \text{ys} = \text{x :: (append2 xs ys)}
\]

\[
\text{val append2} = \text{fn: 'a list} \rightarrow \text{'a list} \rightarrow \text{'a list}
\]

Note: \( \alpha \rightarrow \beta \rightarrow \delta \) means \( \alpha \rightarrow (\beta \rightarrow \delta) \).

- \[
\text{append2} \left[1,2,3\right] \rightarrow \left[8,9\right]
\]

\[
\text{val it} = [1,2,3,8,9] : \text{int list}
\]

- \[
\text{val app123} = \text{append2} \left[1,2,3\right];
\]

\[
\text{val app123} = \text{fn : int list} \rightarrow \text{int list}
\]

- \[
\text{app123} \rightarrow \left[8,9\right];
\]

\[
\text{val it} = [1,2,3,8,9] : \text{int list}
\]
But what if we want to provide the other argument instead, i.e., append [8,9] to its argument?

- here is one way: (the Ada/C/C++/Java way)
  
  ```haskell
  fun appTo89 xs = append2 xs [8,9]
  ```

- here is another: (using a higher-order function)
  
  ```haskell
  val appTo89 = flip append2 [8,9]
  ```

flip is a function which takes a curried function f and returns a function that works like f but takes its arguments in the reverse order. In other words, it “flips” f’s two arguments. We define it on the next slide...
fun flip f y x = f x y

The type of \texttt{flip} is $(\alpha \rightarrow \beta \rightarrow \gamma) \rightarrow \beta \rightarrow \alpha \rightarrow \gamma$. Why?

- Consider $(f \ x)$. \texttt{f} is a function; its parameter must have the same type as \texttt{x}.
  \[
  f : A \rightarrow B \quad x : A \quad (f \ x) : B
  \]

- Now consider $(f \ x \ y)$. Because function application is left-associative, $f \ x \ y \equiv (f \ x) \ y$. Therefore, $(f \ x)$ must be a function, and its parameter must have the same type as \texttt{y}:
  \[
  (f \ x) : C \rightarrow D \quad y : C \quad (f \ x \ y) : D
  \]

- Note that $B$ must be the same as $C \rightarrow D$. We say that $B$ must \textit{unify} with $C \rightarrow D$.

- The return type of \texttt{flip} is whatever the type of $f \ x \ y$ is. After renaming the types, we have the type given at the top.
The type system is defined in terms of inference rules. For example, here is the rule for variables:

\[
(x : \tau) \in E \quad \frac{}{E \vdash x : \tau}
\]

and the one for function calls:

\[
\frac{E \vdash e_1 : \tau' \rightarrow \tau \quad E \vdash e_2 : \tau'}{E \vdash e_1 \ e_2 : \tau}
\]

and here is the rule for \texttt{if} expressions:

\[
\frac{E \vdash e : \text{bool} \quad E \vdash e_1 : \tau \quad E \vdash e_2 : \tau}{E \vdash \text{if} \ e \ \text{then} \ e_1 \ \text{else} \ e_2 : \tau}
\]
- fun exists pred [ ] = false
  | exists pred (x::xs) = pred x orelse
    exists pred xs;

val exists = fn : ('a -> bool) -> 'a list -> bool

- pred is a predicate: a function that returns a boolean
- exists checks whether pred returns true for any member of the list

- exists (fn i => i = 1) [2, 3, 4];
  val it = false : bool
- exists (fn i => i = 1) [2, 3, 4];
val it = false : bool

Now partially apply exists:
- val hasOne = exists (fn i => i = 1);
val hasOne = fn : int list -> bool
- hasOne [3,2,1];
val it = true : bool
fun all pred [ ] = true
    | all pred (x::xs) = pred x andalso all pred xs

fun filter pred [ ] = [ ]
    | filter pred (x::xs) = if pred x
                          then x :: filter pred xs
                          else filter pred xs

all : (α → bool) → α list → bool

filter : (α → bool) → α list → α list
let provides local scope:

(* standard Newton-Raphson *)
fun findroot (a, x, acc) =
  let val nextx = (a / x + x) / 2.0 (* nextx is the next approximation *)
  in
    if abs (x - nextx) < acc * x
    then nextx
    else findroot (a, nextx, acc)
  end
fun mrgSort op< [] = []
| mrgSort op< [x] = [x]
| mrgSort op< (a::bs) = 
  let fun partition (left, right, []) = 
    (left, right) (* done partitioning *)
  | partition (left, right, x::xs) = 
    (* put x to left or right *)
    if x < a 
    then partition (x::left, right, xs)
    else partition (left, x::right, xs)
  val (left, right) = partition ([], [a], bs)
  in
  mrgSort op< left @ mrgSort op< right
end

mrgSort : (α * α → bool) → α list → α list
Another variant of mergesort

fun mrgSort op < [] = []
| mrgSort op < [x] = [x]
| mrgSort op < (a::bs) =
  let fun deposit (x, (left, right)) =
    if x < a
    then (x::left, right)
    else (left, x::right)
  val (left, right) = foldr deposit ([], [a]) bs
in
  mrgSort op < left @ mrgSort op < right
end

mrgSort : (α * α → bool) → α list → α list
The type system

- **primitive types**: `bool, int, char, real, string, unit`
- **constructors**: `list, array, product (tuple), function, record`
- **“datatypes”**: a way to make new types
- **structural equivalence** (except for datatypes)
  - as opposed to name equivalence in e.g., Ada
- an expression has a corresponding type expression
- the interpreter builds the type expression for each input
- type checking requires that type of functions’ parameters match the type of their arguments, and that the type of the context matches the type of the function’s result
Records in ML obey structural equivalence (unlike records in many other languages).

A type declaration: *only needed if you want to refer to this type by name*

```ml
type vec = { x : real, y : real }
```

A variable declaration:

```ml
val v = { x = 2.3, y = 4.1 }
```

Field selection:

```ml
#x v
```

Pattern matching in a function:

```ml
fun dist {x,y} = 
  sqrt (pow (x, 2.0) + pow (y, 2.0))
```
Tuples are actually records.

("I", "Love", "Programming", "Languages")

is actually...

{1="I", 2="Love", 3="Programming", 4="Languages"}

Expression \#2 extracts the second element of the tuple. Or, "Love" above.
A **datatype** declaration:

- defines a new type *that is not equivalent to any other type* (name equivalence)
- introduces *data constructors*
  - *data constructors* can be used in patterns
  - they are also values themselves
datatype tree = Leaf of int
| Node of tree * tree

Tree is a type constructor.

Leaf and Node are data constructors:
- Leaf : int → tree
- Node : tree * tree → tree

We can define functions by pattern matching:

fun sum (Leaf t) = t
| sum (Node (t1, t2)) = sum t1 + sum t2
fun flatten (Leaf t) = [t]
| flatten (Node (t1, t2)) = flatten t1 @ flatten t2

flatten : tree → int list

datatype 'a gentree = Leaf of 'a
| Node of 'a gentree * 'a gentree

val names = Node (Leaf "this", Leaf "that")

names : string gentree
Pattern elements:

- integer literals: 4, 19
- character literals: #'a'
- string literals: "hello"
- data constructors: Node (⋯)
  - depending on type, may have arguments, which would also be patterns
- variables: x, ys
- wildcard: _

Convention is to capitalize data constructors and structure names. Also, start variables and type constructors with lower-case.
More rules of pattern matching

Special forms:

■ (), {} – the unit value
■ [] – empty list
■ [p1, p2, ⋮, pn] means \((p1 :: (p2 :: ⋮ (pn :: [])⋯))\)
■ (p1, p2, ⋮, pn) – a tuple
■ \{field1, field2, ⋮ fieldn\} – a record
■ \{field1, field2, ⋮ fieldn, \ldots\} – a partially specified record using a *wildcard*.
■ \(v\) as \(p\)
  – \(v\) is a name for the entire pattern \(p\)
Example: \(M\) as \(x::xs\) binds \(M\) to the pattern \(x::xs\).
option is a built-in datatype:

```haskell
datatype 'a option = NONE | SOME of 'a
```

Defining a simple lookup function:

```haskell
fun lookup eq key [] = NONE
  | lookup eq key ((k,v)::kvs) = 
    if eq (key, k)
    then SOME v
    else lookup eq key kvs
```

Is the type of `lookup`:

```
(\alpha \times \alpha \to \text{bool}) \to \alpha \to (\alpha \times \beta) \text{list} \to \beta \text{option}
```

No! It's slightly more general:

```
(\alpha_1 \times \alpha_2 \to \text{bool}) \to \alpha_1 \to (\alpha_2 \times \beta) \text{list} \to \beta \text{option}
```
Another lookup function

We don't need to pass two arguments when one will do:

```haskell
fun lookup _ [] = NONE
| lookup checkKey ((k,v):kvs) = 
  if checkKey k
  then SOME v
  else lookup checkKey kvs
```

The type of this lookup:

\[(\alpha \rightarrow \text{bool}) \rightarrow (\alpha \times \beta)\text{list} \rightarrow \beta \text{ option}\]
map: \((\alpha \rightarrow \beta) \rightarrow \alpha\text{ list} \rightarrow \beta\text{ list}\)

\[
\text{map (fn i => i + 1) [7, 15, 3] } \Rightarrow [8, 16, 4]
\]

foldl: \((\alpha \times \beta \rightarrow \beta) \rightarrow \beta \rightarrow \alpha\text{ list} \rightarrow \beta\)

\[
\text{foldl (fn (a,b) => "(" ^ a ^ " +" ^ b ^ ")") "0" ["1", "2", "3"] } \Rightarrow "(3+(2+(1+0)))"
\]

foldr: \((\alpha \times \beta \rightarrow \beta) \rightarrow \beta \rightarrow \alpha\text{ list} \rightarrow \beta\)

\[
\text{foldr (fn (a,b) => "(" ^ a ^ " +" ^ b ^ ")") "0" ["1", "2", "3"] } \Rightarrow "(1+(2+(3+0)))"
\]

filter: \((\alpha \rightarrow \text{bool}) \rightarrow \alpha\text{ list} \rightarrow \alpha\text{ list}\)
Ad hoc overloading interferes with type inference:

\[
\text{fun plus } x \ y = x + y
\]

Operator ‘+’ is overloaded, but types cannot be resolved from context (defaults to int).

We can use explicit typing to select interpretation:

\[
\begin{align*}
\text{fun mix1} \ (x, y, z) &= x * y + z : \text{real} \\
\text{fun mix2} \ (x: \text{real}, y, z) &= x * y + z
\end{align*}
\]
- A function whose type expression has type variables applies to an infinite set of types.
- Equality of type expressions means structural not name equivalence.
- All applications of a polymorphic function use the same body: no need to instantiate.

```ml
let val ints = [1, 2, 3];
    val strs = ["this", "that"];
in
    len ints + (* int list -> int *)
    len strs  (* string list -> int *)
end;
```
An ML signature specifies an interface for a module.

signature STACKS =

sig

  type stack

  exception Underflow

  val empty : stack

  val push : char * stack -> stack

  val pop : stack -> char * stack

  val isEmpty : stack -> bool

end
structure Stacks : STACKS =
struct
  type stack = char list
  exception Underflow
  val empty = []
  val push = op ::
     fun pop (c::cs) = (c, cs)
     | pop [] = raise Underflow
  fun isEmpty [] = true
     | isEmpty _ = false
end