Homework #1: Morlet Wavelets

Due February 16th, 2016
Professor Davi Geiger.

The goal of this assignment is to have your environment set up with the libraries we will be using during the semester. We will have Feb 16th to be a class lab and you are expected to show your system and debug it further if needed.

**Morlet Wavelets (One Peak)**

\[
\psi_{\sigma,\theta}(u) = \frac{C_1}{\sigma} \left( e^{i \frac{\pi}{2\sigma}(u.e_{\theta})} - C_2 \right) e^{-\frac{u^2}{2\sigma^2}} ; \quad u = (x, y), e_{\theta} = (\cos \theta, \sin \theta)
\]

1. Let us first create the Morlet Wavelets, for different parameters,

   (a) \( \sigma = 1 \), (b) \( \sigma = 3 \), (c) \( \sigma = 6 \),

   (i) \( \theta = 0 \), (ii) \( \theta = \frac{\pi}{6} \), (iii) \( \theta = \frac{\pi}{4} \), (iv) \( \theta = \frac{\pi}{3} \), (v) \( \theta = \frac{\pi}{2} \), (vi) \( \theta = \frac{2\pi}{3} \), (vii) \( \theta = \frac{3\pi}{4} \),

   (viii) \( \theta = \frac{5\pi}{6} \)

These are 24 combinations of parameters. Let us refer to the parameters as \( \lambda = (\sigma, \theta) \), i.e., there are \( \lambda_1, ..., \lambda_{24} \) different values. To represent each filter, plot these wavelets, the real part and the complex part using windows of size 37 x 37 pixels (since the largest scale is \( \sigma = 6 \), so the window is 3 x 6=18, i.e., from -18 to 18 \( \rightarrow \) 37 pixels.)

They look like (see next page)
The real part (above) and the complex part (below) of Morlet 2D for $\sigma=1,3,6$ and $\theta$ values are shown on windows of size 37 pixels. The function values are shown as “0” $\rightarrow$ grey, negative numbers $\rightarrow$ dark, positive numbers $\rightarrow$ bright.
2. For each of the two images below,

![Butterfly](image1.png)  ![noisy circle](image2.png)

and for each \( \lambda_i \), perform a convolution using a convolution function (either with the complex wavelet at once, or with the real part of the wavelet and separately with the complex part). Produce the results, i.e., show the “48 images” associated to \( W_{\lambda_i}I(u) \), where

\[
W_{\lambda_i}I(u) = \psi_{\lambda_i} \ast I(u) = \sum_{x'} \sum_{y'} I(x-x',y-y') \psi_{\lambda_i}(x',y') \quad i = 1, \ldots, 24
\]

24 are associated to the convolution with the real part and 24 with the convolution of the complex part or

\[
W_{\lambda_i}^{\text{Real}}I(u) = \psi_{\lambda_i}^{\text{Real}} \ast I(u) = \sum_{x'} \sum_{y'} I(x-x',y-y') \psi_{\lambda_i}^{\text{Real}}(x',y') \quad i = 1, \ldots, 24
\]

\[
W_{\lambda_i}^{\text{Im}}I(u) = \psi_{\lambda_i}^{\text{Im}} \ast I(u) = \sum_{x'} \sum_{y'} I(x-x',y-y') \psi_{\lambda_i}^{\text{Im}}(x',y') \quad i = 1, \ldots, 24
\]

**NOTE:** Again, your wavelets are represented by a 37 x 37 pixels window which you use to perform the convolution with the image.

Lastly, convolve the image with a Gaussian Blur

\[
G_{\sigma=6} \ast I(u) = C e^{-\frac{u^2}{2\sigma^2}} \ast I(u) = \sum_{x'} \sum_{y'} I(x',y') C e^{-\frac{(x'-x)^2+(y'-y)^2}{2\sigma^2}}
\]

This Gaussian function also is described by a 37 x 37 pixels window So all together, we have 49 images.

**DELIVERY:** In class. Please create your own display. Make sure to put captions indicating the parameters associated to the images you display. Also, please, make sure to also show the filters. The Grader/Tutor and myself will see your results in class.