Assume you are given a binary search tree $T$ on $n$ elements. Using a slight modification of the PostOrder-Tree-Walk procedure, argue (by writing the pseudocode!) that in time $\Theta(n)$ you can compute, for every node $v$, the number of nodes (call it $v.less$) in $v$’s sub-tree which are less than $v$.

(Hint: In addition to $v.less$, also compute the total number of nodes in $v$’s subtree.)

(a) (5 points) Start with the simpler case when all the elements are distinct.

Solution: ***************** INSERT YOUR SOLUTION HERE ***************** 

(b) (3 points) Your solution for part (a) will probably not work if some of the elements of $T$ could be the same. Show how to extend the solution for part (a) to handle this case. (Feel free to right away solve this general case for all 5+3=8 points.)

Solution: ***************** INSERT YOUR SOLUTION HERE ***************** 

The Preorder-Tree-Walk of a binary search tree $T$ is: 6, 2, 1, 4, 3, 5, 7, 9, 8. Draw $T$.

Solution: ***************** INSERT YOUR SOLUTION HERE *****************
Prove or show a counterexample for the following statements:¹

(a) (5 points) For any binary search tree $T$ and any element $x \not\in T$, if one applies in sequence $\text{Insert}(x)$ followed by $\text{Delete}(x)$, then the result will be always $T$.

Solution: ***************** INSERT YOUR SOLUTION HERE ************ *

(b) (5 points) For any binary search tree $T$ and any element $x \in T$, if one applies in sequence $\text{Delete}(x)$ followed by $\text{Insert}(x)$, then the result will be always $T$.

Solution: ***************** INSERT YOUR SOLUTION HERE ************ *

¹If the statement is true, you need to give a general argument for any $T$. If false, you choose a specific $T$ which illustrates the problem.
Consider the following 2-3 tree $T$:

```
   9
  /   \
 5     9
 / \
1 3 5 7 8 9
```

(a) (5 points) Show an element $x \notin T$ such that applying in sequence $\text{Insert}(x)$ and $\text{Delete}(x)$ will result with a tree $T'$ that is different from $T$.

Solution: ***************** INSERT YOUR SOLUTION HERE *****************

(b) (5 points) Show an element $x \in T$ such that applying in sequence $\text{Delete}(x)$ and $\text{Insert}(x)$ will result with a tree $T'$ that is different from $T$.

Solution: ***************** INSERT YOUR SOLUTION HERE *****************