Let $A[1,\ldots,n]$ be an array of $n$ distinct numbers. If $i < j$ and $A[i] > A[j]$, then the pair $(i,j)$ is called an inversion of $A$.

(a) (2 points) List the five inversions of the array $(2, 3, 8, 6, 1)$.

Solution: **************** INSERT YOUR SOLUTION HERE ****************

(b) (3 points) Which arrays with elements from the set $\{1,2,\ldots,n\}$ have the smallest and the largest number of inversions and why? State the expressions exactly in terms of $n$.

Solution: **************** INSERT YOUR SOLUTION HERE ****************

(c) (5 points) What is the relationship between the running time of INSERTION_SORT and the number of inversions $I$ in the input array? Justify your answer.

Solution: **************** INSERT YOUR SOLUTION HERE ****************
Let $A[1 \ldots n]$ be an array of pairwise different numbers. We call a pair of indices $1 \leq i < j \leq n$ an inversion of $A$ if $A[i] > A[j]$. The goal of this problem is to develop a divide-and-conquer based algorithm running in time $\Theta(n \log n)$ for computing the number of inversions in $A$.

(a) (8 points) Suppose you are given a pair of sorted integer arrays $A$ and $B$ of length $n/2$ each. Let $C$ an $n$-element array consisting of the concatenation of $A$ followed by $B$. Give an algorithm (in pseudocode) for counting the number of inversions in $C$ and analyze its runtime. Make sure you also argue (in English) why your algorithm is correct.

Solution: ****************** INSERT YOUR SOLUTION HERE ******************

(b) (8 points) Give an algorithm (in pseudocode) for counting the number of inversions in an $n$ element array $A$ that runs in time $\Theta(n \log n)$. Make sure you formally prove that your algorithm runs in time $\Theta(n \log n)$ (e.g., write the recurrence and solve it.)

(Hint: Combine Merge Sort with part (a).)

Solution: ****************** INSERT YOUR SOLUTION HERE ******************
Consider the following recurrence $T(n) = 4T(n/2) + n^2 \log n$, $T(1) = 1$.

(a) (2 points) Can the master’s theorem, as stated in the book, be applied to solve this recurrence? If yes, apply it. If not, formally explain the reason why.

Solution: ***************** INSERT YOUR SOLUTION HERE ************

(b) (4 points) Solve the above recurrence using the recursion tree method.

Solution: ***************** INSERT YOUR SOLUTION HERE ************

(c) (4 points) Formally verify that your answer from part (b) is correct using induction.

Solution: ***************** INSERT YOUR SOLUTION HERE ************

(d) (5 points) Solve the above recurrence exactly using domain-range substitution.

Solution: ***************** INSERT YOUR SOLUTION HERE ************
Solutions to Problem 4 of Homework 2 (5 points)

Solve precisely the recurrence $T(n) = T(\sqrt{n}) + \log n$, with $T(2) = 2$ (assume $n$ is such that $\sqrt{\ldots \sqrt{n}}$ is always an integer). (Hint: Substitute $n = 2^k$ ... Then substitute again.)

Solution: ******************* INSERT YOUR SOLUTION HERE *******************  

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