Search Engine Architecture

6. Link Analysis
Today’s Agenda

• Graph problems and representations
• Parallel breadth-first search
• PageRank
• Optimizing graph algorithms

What’s a graph?

- **G = (V,E)**, where
  - **V** represents the set of vertices (nodes)
  - **E** represents the set of edges (links)
  - Both vertices and edges may contain additional information
- Different types of graphs:
  - Directed vs. undirected edges
  - Presence or absence of cycles
- Graphs are everywhere:
  - Hyperlink structure of the web
  - Physical structure of computers on the Internet
  - Interstate highway system
  - Social networks

Some Graph Problems

- Finding shortest paths
  - Routing Internet traffic and UPS trucks
- Finding minimum spanning trees
  - Telco laying down fiber
- Finding Max Flow
  - Airline scheduling
- Identify “special” nodes and communities
  - Breaking up terrorist cells, spread of avian flu
- Bipartite matching
  - Monster.com, Match.com
- And of course... PageRank

Graphs and MapReduce

• A large class of graph algorithms involve:
  • Performing computations at each node: based on node features, edge features, and local link structure
  • Propagating computations: “traversing” the graph

• Key questions:
  • How do you represent graph data in MapReduce?
  • How do you traverse a graph in MapReduce?

Representing Graphs

• $G = (V, E)$
• Two common representations
  • Adjacency matrix
  • Adjacency list

Adjacency Matrices

Represent a graph as an $n \times n$ square matrix $M$

- $n = |V|$  
- $M_{ij} = 1$ means a link from node $i$ to $j$

\[
\begin{array}{cccc}
  & 1 & 2 & 3 & 4 \\
 1 & 0 & 1 & 0 & 1 \\
 2 & 1 & 0 & 1 & 1 \\
 3 & 1 & 0 & 0 & 0 \\
 4 & 1 & 0 & 1 & 0 \\
\end{array}
\]

Adjacency Matrices: Critique

• **Advantages:**
  • Amenable to mathematical manipulation
  • Iteration over rows and columns corresponds to computations on outlinks and inlinks

• **Disadvantages:**
  • Lots of zeros for sparse matrices
  • Lots of wasted space

Adjacency Lists

Take adjacency matrices... and throw away all the zeros

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
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<tr>
<td>2</td>
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<td>1</td>
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<tr>
<td>3</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

1: 2, 4
2: 1, 3, 4
3: 1
4: 1, 3

Adjacency Lists: Critique

• Advantages:
  • Much more compact representation
  • Easy to compute over outlinks

• Disadvantages:
  • Much more difficult to compute over inlinks

Single-Source Shortest Path

- **Problem**: find shortest path from a source node to one or more target nodes
  - Shortest might also mean lowest weight or cost
- First, a refresher: Dijkstra’s Algorithm
Dijkstra’s Algorithm Example

Example from CLR
Dijkstra’s Algorithm Example

Example from CLR
Dijkstra’s Algorithm Example

Example from CLR
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Example from CLR
Single-Source Shortest Path

- **Problem:** find shortest path from a source node to one or more target nodes
  - Shortest might also mean lowest weight or cost
- Single processor machine: Dijkstra’s Algorithm
- MapReduce: parallel breadth-first search (BFS)

Finding the Shortest Path

- Consider simple case of equal edge weights
- Solution to the problem can be defined inductively
- Here's the intuition:
  - Define: $b$ is reachable from $a$ if $b$ is on adjacency list of $a$
    \[
    \text{DISTANCETo}(s) = 0
    \]
  - For all nodes $p$ reachable from $s$, \( \text{DISTANCETo}(p) = 1 \)
  - For all nodes $n$ reachable from some other set of nodes $M$, \( \text{DISTANCETo}(n) = 1 + \min(\text{DISTANCETo}(m), m \in M) \)

Visualizing Parallel BFS

From Intuition to Algorithm

• Data representation:
  • Key: node \( n \)
  • Value: \( d \) (distance from start), adjacency list (nodes reachable from \( n \))
  • Initialization: for all nodes except for start node, \( d = \infty \)

• Mapper:
  • \( \forall m \in \text{adjacency list}: \text{emit } (m, d + 1) \)

• Sort/Shuffle
  • Groups distances by reachable nodes

• Reducer:
  • Selects minimum distance path for each reachable node
  • Additional bookkeeping needed to keep track of actual path

Multiple Iterations Needed

• Each MapReduce iteration advances the “frontier” by one hop
  • Subsequent iterations include more and more reachable nodes as frontier expands
  • Multiple iterations are needed to explore entire graph

• Preserving graph structure:
  • Problem: Where did the adjacency list go?
  • Solution: mapper emits \((n, \text{adjacency list})\) as well

BFS Pseudo-Code

1: class Mapper
2:   method Map(nid n, node N)
3:     d ← N.Distance
4:     Emit(nid n, N) ▷ Pass along graph structure
5:     for all nodeid m ∈ N.AdjacencyList do
6:         Emit(nid m, d + 1) ▷ Emit distances to reachable nodes

1: class Reducer
2:   method Reduce(nid m, [d₁,d₂,...])
3:     d_min ← ∞
4:     M ← ∅
5:     for all d ∈ counts [d₁,d₂,...] do
6:       if IsNode(d) then
7:         M ← d ▷ Recover graph structure
8:       else if d < d_min then
9:         d_min ← d ▷ Look for shorter distance
10:        M.Distance ← d_min
11:       Emit(nid m, node M) ▷ Update shortest distance

Stopping Criterion

• How many iterations are needed in parallel BFS (equal edge weight case)?
• Convince yourself: when a node is first “discovered”, we’ve found the shortest path
• Now answer the question...
  • Six degrees of separation?
• Practicalities of implementation in MapReduce

Comparison to Dijkstra

• Dijkstra’s algorithm is more efficient
  • At each step, only pursues edges from minimum-cost path inside frontier

• MapReduce explores all paths in parallel
  • Lots of “waste”
  • Useful work is only done at the “frontier”

• Why can’t we do better using MapReduce?

Single Source: Weighted Edges

- Now add positive weights to the edges
  - Why can’t edge weights be negative?
- Simple change: add weight $w$ for each edge in adjacency list
  - In mapper, emit $(m, d + w_p)$ instead of $(m, d + 1)$ for each node $m$
- That’s it?

Stopping Criterion

• How many iterations are needed in parallel BFS (positive edge weight case)?

• Convince yourself: when a node is first “discovered”, we’ve found the shortest path

Stopping Criterion

- How many iterations are needed in parallel BFS (positive edge weight case)?
- Convince yourself: when a node is first “discovered”, we’ve found the shortest path

Not true!

Additional Complexities

Stopping Criterion

- How many iterations are needed in parallel BFS (positive edge weight case)?
- Practicalities of implementation in MapReduce

PageRank
Graphs and MapReduce

- A large class of graph algorithms involve:
  - Performing computations at each node: based on node features, edge features, and local link structure
  - Propagating computations: “traversing” the graph
- Generic recipe:
  - Represent graphs as adjacency lists
  - Perform local computations in mapper
  - Pass along partial results via outlinks, keyed by destination node
  - Perform aggregation in reducer on inlinks to a node
  - Iterate until convergence: controlled by external “driver”
  - Don’t forget to pass the graph structure between iterations

Random Walks Over the Web

- Random surfer model:
  - User starts at a random Web page
  - User randomly clicks on links, surfing from page to page
- PageRank
  - Characterizes the amount of time spent on any given page
  - Mathematically, a probability distribution over pages
- PageRank captures notions of page importance
  - Correspondence to human intuition?
  - One of thousands of features used in web search (query-independent)

PageRank: Defined

Given page \( x \) with inlinks \( t_1 \ldots t_n \), where

- \( C(t) \) is the out-degree of \( t \)
- \( \alpha \) is probability of random jump
- \( N \) is the total number of nodes in the graph

\[
PR(x) = \alpha \left( \frac{1}{N} \right) + (1 - \alpha) \sum_{i=1}^{n} \frac{PR(t_i)}{C(t_i)}
\]

Computing PageRank

- Properties of PageRank
  - Can be computed iteratively
  - Effects at each iteration are local
- Sketch of algorithm:
  - Start with seed $PR_i$ values
  - Each page distributes $PR_i$ “credit” to all pages it links to
  - Each target page adds up “credit” from multiple in-bound links to compute $PR_{i+1}$
  - Iterate until values converge

Simplified PageRank

• First, tackle the simple case:
  • No random jump factor
  • No dangling nodes
• Then, factor in these complexities...
  • Why do we need the random jump?
  • Where do dangling nodes come from?

Sample PageRank Iteration (1)

Sample PageRank Iteration (2)

Iteration 2

Map

Reduce

MapReduce

n_1 \[ n_2, n_4 \]  n_2 \[ n_3, n_5 \]  n_3 \[ n_4 \]  n_4 \[ n_5 \]  n_5 \[ n_1, n_2, n_3 \]

n_1 \[ n_2, n_4 \]  n_2 \[ n_3, n_5 \]  n_3 \[ n_4 \]  n_4 \[ n_5 \]  n_5 \[ n_1, n_2, n_3 \]

n_1 (0.1)  n_2 (0.133)  n_3 (0.183)  n_4 (0.2)  n_5 (0.383)
PageRank Pseudo-Code

1: class Mapper
2:   method Map(nid n, node N)
3:     \( p \leftarrow \frac{N.\text{PageRank}}{|N.\text{AdjacencyList}|} \)
4:     Emit(nid n, N) \quad \triangleright \text{Pass along graph structure}
5:     for all nodeid \( m \in N.\text{AdjacencyList} \) do
6:       Emit(nid m, p) \quad \triangleright \text{Pass PageRank mass to neighbors}

1: class Reducer
2:   method Reduce(nid m, [\( p_1, p_2, \ldots \)])
3:     \( M \leftarrow \emptyset \)
4:     for all \( p \in \text{counts} \) \( [p_1, p_2, \ldots] \) do
5:       if IsNode(p) then
6:         \( M \leftarrow p \) \quad \triangleright \text{Recover graph structure}
7:       else
8:         \( s \leftarrow s + p \) \quad \triangleright \text{Sums incoming PageRank contributions}
9:     \( M.\text{PageRank} \leftarrow s \)
10:    Emit(nid m, node M)

Complete PageRank

- Two additional complexities
  - What is the proper treatment of dangling nodes?
  - How do we factor in the random jump factor?
- Solution:
  - Second pass to redistribute “missing PageRank mass” and account for random jumps

\[
p' = \alpha \left(\frac{1}{N}\right) + (1 - \alpha) \left(\frac{m}{N} + p\right)
\]

- \( p \) is PageRank value from before, \( p' \) is updated PageRank value
- \( N \) is the number of nodes in the graph
- \( m \) is the missing PageRank mass
- Additional optimization: make it a single pass!

PageRank Convergence

- Alternative convergence criteria
  - Iterate until PageRank values don’t change
  - Iterate until PageRank rankings don’t change
  - Fixed number of iterations
- Convergence for web graphs?
  - Not a straightforward question
- Watch out for link spam:
  - Link farms
  - Spider traps
  - ...

Beyond PageRank

- Variations of PageRank
  - Weighted edges
  - Personalized PageRank
- Variants on graph random walks
  - Hubs and authorities (HITS)
  - SALSA

Applications

- Static prior for web ranking
- Identification of “special nodes” in a network
- Link recommendation
- Additional feature in any machine learning problem

Other Classes of Graph Algorithms

- Subgraph pattern matching
- Computing simple graph statistics
  - Degree vertex distributions
- Computing more complex graph statistics
  - Clustering coefficients
  - Counting triangles

MapReduce Sucks

- Needless graph shuffling
- Checkpointing at each iteration

Iterative Algorithms
MapReduce sucks at iterative algorithms

- Alternative programming models (later)
- Easy fixes (now)

In-Mapper Combining

- Use combiners
  - Perform local aggregation on map output
  - Downside: intermediate data is still materialized
- Better: in-mapper combining
  - Preserve state across multiple map calls, aggregate messages in buffer, emit buffer contents at end
  - Downside: requires memory management

Better Partitioning

• Default: hash partitioning
  • Randomly assign nodes to partitions

• Observation: many graphs exhibit local structure
  • E.g., communities in social networks
  • Better partitioning creates more opportunities for local aggregation

• Unfortunately, partitioning is **hard!**
  • Sometimes, chicken-and-egg...
  • But cheap heuristics sometimes available
  • For webgraphs: range partition on domain-sorted URLs

Schimmy Design Pattern

• Basic implementation contains two dataflows:
  • Messages (actual computations)
  • Graph structure ("bookkeeping")
• Schimmy: separate the two dataflows, shuffle only the messages
• Basic idea: merge join between graph structure and messages

Schimmy Design Pattern

- Basic implementation contains two dataflows:
  - Messages (actual computations)
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  - Basic idea: merge join between graph structure and messages

Do the Schimmy!

- Schimmy = reduce side parallel merge join between graph structure and messages
- Consistent partitioning between input and intermediate data
- Mappers emit only messages (actual computation)
- Reducers read graph structure directly from HDFS

Experiments

- Cluster setup:
  - 10 workers, each 2 cores (3.2 GHz Xeon), 4GB RAM, 367 GB disk
  - Hadoop 0.20.0 on RHELS 5.3

- Dataset:
  - First English segment of ClueWeb09 collection
  - 50.2m web pages (1.53 TB uncompressed, 247 GB compressed)
  - Extracted webgraph: 1.4 billion links, 7.0 GB
  - Dataset arranged in crawl order

- Setup:
  - Measured per-iteration running time (5 iterations)
  - 100 partitions

Results

Questions?