Max flow exercises

Exercise 1. Software Allocation - UVa 259

A computing center has ten different computers (numbered 0 to 9) on which applications can run. The computers are not multi-tasking, so each machine can run only one application at any time. There are 26 applications, named A to Z. Whether an application can run on a particular computer can be found in a job description (see below).

Every morning, the users bring in their applications for that day. It is possible that two users bring in the same application; in that case two different, independent computers will be allocated for that application.

A clerk collects the applications, and for each different application he makes a list of computers on which the application could run. Then, he assigns each application to a computer. Remember: the computers are not multi-tasking, so each computer must handle at most one application in total. (An application takes a day to complete, so that sequencing i.e. one application after another on the same machine is not possible.)

Output a possible matching between computers and applications, or ! if such a matching is not possible.

<table>
<thead>
<tr>
<th>Sample input</th>
<th>Sample output</th>
</tr>
</thead>
<tbody>
<tr>
<td>A4 01234</td>
<td>AAAA_QPPPP</td>
</tr>
<tr>
<td>Q1 5</td>
<td>!</td>
</tr>
<tr>
<td>P4 56789;</td>
<td></td>
</tr>
<tr>
<td>A4 01234;</td>
<td></td>
</tr>
<tr>
<td>Q1 5;</td>
<td></td>
</tr>
<tr>
<td>P5 56789;</td>
<td></td>
</tr>
</tbody>
</table>

Exercise 2. Clever Naming Patterns - UVa 11418

Piotr is organizing a programming competition consisting of $n$ problems, numbered $A$, $B$, $C$, etc. He wants to name the first problem starting with the letter $A$, the second problem starting with the letter $B$, and so on. However, he cannot simply come up with random words for problem titles - that wouldn’t make sense. For each problem, he has come up with a list of acceptable names. Help Piotr pick an ordering of the problems and an acceptable name for each one so that each problem’s name starts with the correct letter of the alphabet.

Construct the max flow graph for the following example.
<table>
<thead>
<tr>
<th>Problem</th>
<th>Possible names</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Argonaut, Bouncy, Delicate, Cavernous</td>
</tr>
<tr>
<td>2</td>
<td>Appalachian, Breakfast, Crispy</td>
</tr>
<tr>
<td>3</td>
<td>Autumn, Buckwheat</td>
</tr>
<tr>
<td>4</td>
<td>Askew</td>
</tr>
</tbody>
</table>

**Exercise 3. Councilling - UVa 10511**

Each resident of a particular town is a member of zero or more clubs and also a member of exactly one political party. Each club is to nominate one of its members to represent it on the town council so that the number of council members belonging to any given party does not equal or exceed half the membership of the council. The same person may not represent two clubs; that is there must be a 1-1 relationship between clubs and council members. Your job is to select the council members subject to these constraints.

Construct the max flow graph for the following example.

<table>
<thead>
<tr>
<th>Person</th>
<th>Club Memberships</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fred</td>
<td>Jets, Sharks, Knights</td>
</tr>
<tr>
<td>John</td>
<td>Rooks, Jets, Rotary</td>
</tr>
<tr>
<td>Mary</td>
<td>Rooks, Knights, Rotary</td>
</tr>
<tr>
<td>Ruth</td>
<td>Platypus</td>
</tr>
</tbody>
</table>

**Exercise 4. Sabotage - UVa 10480**

The regime of a small but wealthy dictatorship has been abruptly overthrown by an unexpected rebellion. Because of the enormous disturbances this is causing in world economy, an imperialist military super power has decided to invade the country and reinstall the old regime.

For this operation to be successful, communication between the capital and the largest city must be completely cut. This is a difficult task, since all cities in the country are connected by a computer network using the Internet Protocol, which allows messages to take any path through the network. Because of this, the network must be completely split in two parts, with the capital in one part and the largest city in the other, and with no connections between the parts.

There are large differences in the costs of sabotaging different connections, since some are much more easy to get to than others. So, each link in the network is assigned a cost for it to be cut.

How does finding the max flow of the network help you determine which of the connections to cut?