Numerical Computing - Practice Homework 10

1. Suppose you use Simpson’s rule with \( n \) intervals, and again with \( 2n \) intervals, to approximate a smooth integral on an interval \([a, b]\). The leading term in the error for the composite Simpson’s rule is \( h^4(b - a)f^{(4)}(\eta)/180 \). What is the formula to use Richardson extrapolation to improve your answer by combining these two approximations.

2. The midpoint rule approximates

\[
\int_a^b f(x) \, dx = (b - a)f\left(\frac{(a + b)}{2}\right) \tag{1}
\]

In class we derived the leading term of the error using Taylor series, getting

\[
\int_a^b f(x) \, dx - (b - a)f\left(\frac{(a + b)}{2}\right) = \frac{(b - a)^3}{24}f''\left(\frac{(a + b)}{2}\right) \tag{2}
\]

Use this to derive an error formula for the composite midpoint rule with \( n \) intervals and points \( a = x_0 < x_1 < \cdots < x_n = b \).

3. Suppose the composite midpoint rule is applied to the function \( f(x) = \cos(x^2) \), on the interval \([0, \pi]\). Using your error formula, if \( h = .1 \), what is an upper bound for the worst the error can be? (You can use matlab to plot some things to help you come up with a good error bound).

4. Use the method of undetermined coefficients to find the most accurate quadrature formula to compute \( \int_0^1 f(x) \, dx \) using the points 0, \( 1/3 \), \( 2/3 \), 1.