1. Construct an explicit basis for the lattice \( \{ x \in \mathbb{Z}^n : x_1 + \sum_{i=2}^{n} a_ix_i \equiv 0 \pmod{p} \} \), where \( a_i \in \mathbb{Z}_p \), \( p \) a prime.

2. Exercise 1 in Lecture 1.

3. Take \( a_1, \ldots, a_n \in \mathbb{N} \). The greatest common divisor of \( a_1, \ldots, a_n \), denoted \( \gcd(a_1, \ldots, a_n) \), is the largest integer \( d \) such that \( d \mid a_i \) (meaning \( d \) divides \( a_i \)), for all \( i \in [n] \). Show that there exists \( z_1, \ldots, z_n \in \mathbb{Z} \) such that \( \gcd(a_1, \ldots, a_n) = \sum_{i=1}^{n} z_ia_i \). Give a simple algorithm to compute \( \gcd(a_1, \ldots, a_n) \) (no need to analyze its complexity).