The following assignments are due on March 11 at 4:00 pm. Turn in a listing of your MATLAB programs as well as output from selected runs as well as a discussion. If you wish, you can instead use any other standard programming language.

1. Problem 12.6 in Iserles’ book; note that TST matrices are defined on page 264.

2. Problem 12.10 in Iserles’ book. Note that compatible and consistent ordering means the same thing.

3. Develop, analyze, and program an alternative to the nested dissection algorithm, which was discussed in the lecture on February 7. Consider a simple five point approximation of Poisson’s equation on a square. Introduce separator sets in one direction; each of them consists of a full set of mesh points and they separate the mesh into rectangular parts. The unknowns on the separator sets are eliminated after that all the unknowns of the rectangular areas have been eliminated. The problems on these rectangles are independent of each other and they are handled by band-Cholesky.

   (a) Estimate the cost of using this ordering of the variables when using Cholesky’s algorithm for the subproblems. How does the work estimate depend on the number of separator sets which, of course, then will affect the width of the rectangular strips and the band width for those subproblems? What can be said about the sparsity of the problem that remains after that all the unknowns interior of those rectangles have been eliminated? How can we best take advantage of that sparsity?

   (b) How does such a method compare with band-Cholesky without any separator sets and with the nested dissection ordering of George?
4. In the lecture on February 21, it was shown that a certain tridiagonal matrix with elements that are obtained successively step by step when using the conjugate gradient method, contains approximate information on the extreme eigenvalues of the matrix of the linear system that we are solving.

(a) Suppose that we use the standard form of the conjugate gradient method as on page 316-317 in the textbook. How can we then extract the elements of that tridiagonal matrix?

(b) Implement the conjugate gradient method and use it to solve the linear system of equations that arises from the five-point approximation of $-\Delta$. Study the convergence rate as a function of the number of mesh points.

(c) While running the iteration, generate the tridiagonal matrix and use a Matlab routine to compute the largest and smallest eigenvalues of it. Compare the resulting estimated condition number obtained in this way with the exact value that can be obtained by using Fourier series.

(d) If time allows, try to implement an incomplete Cholesky method; cf. page 254 in the textbook. Use the resulting matrix as a preconditioner in a preconditioned conjugate gradient method.