Buying a Pair of Jeans

A. Statement of the Variables

An agent wants to buy a pair of jeans and there are n different stores to buy. Say for simplicity, there is one pair of jeans per brand, so jeans and brands are being represented by the same variable. How to choose? Which variables play a role in making such decision and how a decision is made? Let us refer to the variable \( x = 1, \ldots, n \) for each pair of Jeans. Say each agent is represented by the variable \( i \). Say the agent must decide if will purchase or not the pair of Jeans, i.e., we need a binary variable \( \text{Purchase}(i, x) = 0, 1 \) to represent such decision. There is also a variable \( \text{Fash}(i, x) \) that represents how much agent \( i \) thinks brand \( x \) is fashionable. Fashionable is the result of many different considerations by agent \( i \) and we need to model it.

. Wallet. How much money to spend and the price of the pair of jeans in each store? So say the cost is \( \text{Price}(x) \) and the budget to buy a pair of Jeans per agent is \( \text{Budget}(i) \). Of Course, \( \text{Price}(x) \leq \text{Budget}(i) \), for agent \( i \) to be able to buy pair of jeans \( x \). Also, it is clear that the lower the price the more agent \( i \) is willing to buy it (sometimes agents may prefer to buy if it is more expensive).

Like. Agent \( i \) must like a pair of jeans \( x \) to want to buy it. “Like” may involve Fit, Trust on a brand and how Fashionable is a pair of Jeans. “Like” maybe modeled as a discrete variable with some states, say “don’t Like”, “maybe Like”, “Like”, “Like a lot”, “love it” so \( \text{Like}(i, x) \in \{1, 2, 3, 4, 5\} \).

. Fit. Agent \( i \) must think the pair of Jeans fit well. This is a somewhat subjective perception, but one that agent \( i \) knows how evaluate. Let us refer to \( \text{Fit}(i, x) \) as a variable in \([0,1]\), where \( \text{Fit}=1 \) means perfect fit and 0 means no chance of fitting.

. Trust. Agent \( i \) must have a sense to trust or not a jeans \( x \). Maybe agent \( i \) bought in the past a jeans \( x \) from the same brand. Here we are assuming that \( x \) is brand as well as the unit. Maybe agent \( i \) heard that a particular brand does not last long or the other way around. So we may consider a variable \( \text{Trust}(i, x) \in [0,1] \), to represent how much the customer believes the brand of \( x \) is to be trusted, it is a good quality Jeans. This reflect the agent \( i \) perception of the history of the brand \( x \).

. Fashion. Agent \( i \) have a sense of fashion. Fashion may include the color of the pair of Jeans \( x \), and many other important details, such as (i) “Washing” types, (ii) “Waist-height” such as low, regular, high, (iii) the cut/style (boot, straight, flair, cigarette, skinny), (iv) color (light, grey, blue, …),(v) ragged, (vi) pocket styles. Fashion may also be acquired by other factors, such as advertising and other agents around agent \( i \). This sense of fashion influences agent \( i \) to prefer one pair of jeans over the other. Let us refer to the variable \( \text{Fash}(i, x) \) which is the fashion perception of agent \( i \) about jeans \( x \). So Fashion maybe
modeled as a discrete variable with some states, say “not fashionable”, “maybe fashionable” “fashionable”, “very fashionable”, “fantastic” so $Fash(i,x) \in \{1,2,3,4,5\}$.

.Advertising. This measures how much advertising is hitting a customer. Advertising is capable of making a pair of jeans $x$ to become more fashionable and/or more trustable. Advertising maybe able to make an item that is otherwise $Fash(i,x) = 1$ to become $Fash(i,x)=2$, for example. $Adv(i,x)$ maybe thought as an external force working on the agent to change fashion states.

.Social Behavior. Every agent has its community for a given subject. The subject here is buying a pair of jeans. Agent $i$ may have its close friends, a set $N_i$ of agents to whom to consult. It may also have some other friends, not so close, to consult about it. They all contribute for agent $i$ to have a fashion sense.

B. Graphical Model

B1. One Agent

Probabilities, States and Variables

- $Purchase_i_t (A,B,\ldots,X)$
- $Like_i_t (A,B,\ldots,X)$
- $Advertise_i_t (A,B,\ldots,X)$
- $Fash_i_t (A,B,\ldots,X)$
- $Fit_i (A,B,\ldots,X)$
- $Trust_i (A,B,\ldots,X)$
- $Budget_i$
- $Price(A,B,\ldots,X)$

Diagram of the graphical model showing the relationships between variables such as purchase, like, advertise, and various attributes like fit, trust, price, and budget.
B2. Many Connected Agents

Probabilities, States and Variables

- Purchase\_i\_t(A,B,...X)
- Like\_i\_t(A,B,...X)
- Advertise\_i\_t(A,B,...X)
- Fash\_i\_t(A,B,...X)
- Fit\_i(A,B,...X)
- Trust\_i(A,B,...X)
- Budget\_i
- Price(A,B,...X)

Social Behavior (Fash)
C. Purchase Modeling it

. Wallet and Purchase. Let us say that the lower the price of the pair of jeans \( x \) the more likely agent \( i \) is to buy it. So a formula for this is simply

\[
P_i(Purchase(i, x) = 1|Price(x)) = e^{-\rho_i \left( \frac{Price(x)}{Budget(i) - Price(x)} \right)^{P_i}}, \quad (1)
\]

where \( \lambda_i \) and \( \Lambda_i \) must be calculated and \( 0 \leq Price(x) \leq Budget(i) \).

It is clear that

\[
P_i(Purchase(i, x) = 0|Price(x)) = 1 - e^{-\rho_i \left( \frac{Price(x)}{Budget(i) - Price(x)} \right)^{P_i}} \quad (2)
\]

BuyJeans.xlsx

. Likeable and Purchase. Likeable makes agent \( i \) more likely to buy \( x \), and can be modeled using the formula

\[
P_i(Purchase(x) = 1|Like_i(x)) = \frac{1}{Z_L} e^{-\Lambda_i |5 - Like_i(x)|^\Lambda_i} \quad (3)
\]

and the coefficients \( (\lambda_i, \Lambda_i, Z) \) must be calculated. It is clear that

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. Like: Fit, Trust, Fashion. Which elements makes agent \( i \) more likely to think that \( x \) is Likeable?

\[
P_{i,x}(Like_i(x)|Fit_i(x), Trust_i(x), Fit(x)) = \frac{1}{Z_{Like}} \left( P_{Trust}(Like_i(x)|Trust(x)) \right)^T \left( P_{Fit}(Like_i(x)|Fit(x)) \right)^F \left( P_{Fash}(Like_i(x)|Fash_i(x)) \right)^{Fa} \quad (4)
\]

where the parameters \( T, F, Fa \) need to be estimated and the normalization constant is

\[
Z_{Like} = \sum_{Like_i=1}^5 \left( P_{Trust}(Like_i(x)|Trust(x)) \right)^T \left( P_{Fit}(Like_i(x)|Fit(x)) \right)^F \left( P_{Fash}(Like_i(x)|Fash_i(x)) \right)^{Fa} \quad (5)
\]

A possible model for \( P_{Fash}(i, x) \) to favor Fashion states that are given by how much agent \( i \) likes \( x \).

\[
P_{Fash}(Like(t)|Fash(t)) = \frac{1}{Z_{Fa}} e^{-\gamma_i |Like(i, x) - Fash(i, x)|^{F_i}} \quad (6)
\]
where \( Z_{Fa} = \sum_{Like=1}^{5} e^{-\gamma_{i} |Like(i,x) - Fash(i,x)|^2} \). Similarly, let us use a model for “Like” given Fit and “Like” given Trust, remembering that a perfect Fit =1 and the full Trust =1 should imply Like = 5 while Fit =0 and the lack of Trust =0 should imply Like = 1

\[
P_{Trust}(Like(t)|Trust) = \frac{1}{Z_T} e^{-\tau_{i} |Like(i,x)-(4+Trust+1)|^2} \tag{7}
\]

\[
P_{Fit}(Like(t)|Fit) = \frac{1}{Z_{Fa}} e^{-\phi_{i} |Like(i,x)-(4+Fit+1)|^2} \tag{8}
\]

Now we must introduce time, to describe the result of the action of forces on agents. The result is to have agents to change Fashion states.

Advertising acts as a force, to increase Fashionable strength to brand x. The following formula may characterize such operation

\[
P_{i}(Fash(i,x,t)|Adv(i,x,t), Fash(i,x,t-1)) = \frac{1}{Z_{Adv}} e^{-Adv(i,x,t)|Fash(i,x,t) - min(Fash(i,x,t-1)+1.5)|^{A_{lx}}} \tag{9}
\]

It suggests that the more advertising the more likely is for the state \( Fash(i,x,t) \) to increase one unit, unless it is already at the highest state, \( Fash(i,x,t) = 4 \). It is as likely to increase two states as it is to stay at the same state, again up to the upper and lower bounds states. Advertising is unlikely to have a negative effect of decreasing one state (it is the same strength as increasing three states in Fashionable, up to the upper and lower bounds states). The larger is \( Adv(i,x,t) \) the more these effects. The parameter \( A_{lx} \) must be calculated. It is clear that

\[
Z_{Adv} = \sum_{Fash(i,x,t)=1}^{5} e^{-Adv(i,x,t)|Fash(i,x,t) - min(Fash(i,x,t-1)+1.5)|^{A_{lx}}} \tag{10}
\]
.Social Behavior.

**Strong Ties.** A close community of agent $i$ is represented by $j \in N_i$ and each $j$ push each member to share their Fashion values. This may be described by

$$P_{ij} (\text{Fash}(i, x, t) | \text{Fash}(j, x, t-1)) = \frac{1}{Z_{S_{ij}}} e^{-\alpha_{ij} |\text{Fash}(i,x,t) - \text{Fash}(j,x,t-1)|^{A_{ij}}}$$  \hspace{1cm} (11)

Note that here we may be considering how agent $j$ “thinks” jeans $x$ will “look” at agent $i$, and not on itself. For example, “fit” is factored on how agent $j$ evaluates $\text{Fash}(j,x,t)$, so it must be using the probability of fitting of jeans $x$ to agent $i$. While both close friends influence each other to get to the same fashion opinion on Jeans $x$, their influence does not have to be symmetric, i.e., $\alpha_{ij} \neq \alpha_{ji}$. In order to normalize (11), we must have

$$Z_{S_{ij}} = \sum_{\text{Fashion}(i,x,t) = 1}^{5} e^{-\alpha_{ij} |\text{Fash}(i,x,t) - \text{Fash}(j,x,t-1)|^{A_{ij}}}$$  \hspace{1cm} (12)

**Weak Ties.** An acquaintance or expert in Fashion may influence agent $i$ in making his choices. This may be modeled as a “weak” tie. Let us refer to it as expert or agent $k$ not in $N_i$.

$$P_{ik} (\text{Fash}(i, x, t) | \text{Fash}(k, x, t)) = \frac{1}{Z_{W_{ik}}} e^{-\beta_{ij} |\text{Fash}(i,x,t) - \text{Fash}(k,x,t)|^{B_{ik}}}$$  \hspace{1cm} (13)

where in this case, agent $k$ is not influenced by agent $i$. To normalize (13)

$$Z_{W_{ik}} (\text{Fash}(k, x, t)) = \sum_{\text{Fashion}(i,x,t) = 1}^{5} e^{-\beta_{ij} |\text{Fash}(i,x,t) - \text{Fash}(k,x,t)|^{B_{ik}}}$$  \hspace{1cm} (14)

It is similar to Advertising in that both are external forces. They act differently, since Advertising pushes the brand up while the weak tie pushes the brand to wherever the weak tie is. The parameter $\beta_{ij}$ must be calculated.
D. Final Model: combining all probabilities

\[
P_{t,x}(\text{Purchase} | \text{Trust}, \text{Fit}, \text{Price}, \overline{\text{Adv}})
= \sum_{\text{Like}(t)=1}^{5} \sum_{\text{Fash}(t)=1}^{5} P_{t,x}(\text{Purchase}, \text{Like}(t), \text{Fash}(t) | \text{Trust}, \text{Fit}, \text{Price}, \overline{\text{Adv}})
= \sum_{\text{Like}(t)=1}^{5} \sum_{\text{Fash}(t)=1}^{5} \left[ P_{t,x}(\text{Purchase} | \text{Like}(t), \text{Fash}(t), \text{Trust}, \text{Fit}, \text{Price}, \overline{\text{Adv}}) \right.
\times P_{t,x}(\text{Like}(t), \text{Fash}(t) | \text{Trust}, \text{Fit}, \text{Price}, \overline{\text{Adv}}) \left. \right]
= \sum_{\text{Like}(t)=1}^{5} \sum_{\text{Fash}(t)=1}^{5} \left[ P_{t,x}(\text{Purchase} | \text{Like}(t), \text{Price}) \times P_{t,x}(\text{Like}(t), \text{Fash}(t) | \text{Trust}, \text{Fit}, \overline{\text{Adv}}) \right]
= \sum_{\text{Like}(t)=1}^{5} \left[ P_{t,x}(\text{Purchase} | \text{Like}(t), \text{Price}) \sum_{\text{Fash}(t)=1}^{5} P_{t,x}(\text{Like}(t) | \text{Fash}(t), \text{Trust}, \text{Fit}, \overline{\text{Adv}}) \right.
\times P_{t,x}(\text{Fash}(t) | \text{Trust}, \text{Fit}, \overline{\text{Adv}}) \left. \right]
\]

Where \( \overline{\text{Adv}} = (\text{Adv}(0), \text{Adv}(1), ..., \text{Adv}(t)) \) is a set of Advertisements accumulated up to the time “t” where the effect of it is being considered. We also consider the approximation

\[
P_{t,x}(\text{Like}(t) | \text{Fash}(t), \text{Trust}, \text{Fit}, \text{Price}, \overline{\text{Adv}}) = P_{t,x}(\text{Like}(t) | \text{Fash}(t), \text{Trust}, \text{Fit})
\]

since (i) Price does not affect how much one likes a pair of jeans \( x \), and (ii) Advertising will affect the sense of Fashion and that will affect how much one likes it, but given Fashion, Advertising does not affect further how much one likes a pair of Jeans \( x \). We further consider from (4)

\[
P_{t,x}(\text{Like}(x) | \text{Fash}(x), \text{Trust}(x), \text{Fit}(x)) = \frac{1}{Z_{\text{Like}}} P_{t,x}(\text{Like}(x) | \text{Fash}(x), \text{Trust}(x), \text{Fit}(x))
= \frac{1}{Z_{\text{Like}}} \left( P_{\text{Trust}}(\text{Like}(x) | \text{Trust}(x)) \right)^T \left( P_{\text{Fit}}(\text{Like}(x) | \text{Fit}(x)) \right)^F \left( P_{\text{Fash}}(\text{Like}(x) | \text{Fash}(x)) \right)^{F_a}
\]

We also consider \( P_{t,x}(\text{Fash}(t) | \text{Fit}, \text{Trust}, \text{Price}, \overline{\text{Adv}}(t)) = P_{t,x}(\text{Fash}(t) | \overline{\text{Adv}}(t)) \) since neither of the other variable affect the Fashion variable. Note that Fit and Trust will affect “Like”, i.e., Like will incorporate fashion as well as trust as well as Fit. So our model (15) can be written as

\[
P_{t,x}(\text{Purchase} | \text{Trust}, \text{Fit}, \text{Price}, \overline{\text{Adv}}) = \\
= P_{t,x}(\text{Purchase} | \text{Price})
\times \sum_{\text{Like}(t)=1}^{5} \left\{ P_{t,x}(\text{Purchase} | \text{Like}(t)) \right. \\
\times \frac{1}{Z_{\text{Like}}} \left( P_{\text{Trust}}(\text{Like}(x) | \text{Trust}(x)) \right)^T \left( P_{\text{Fit}}(\text{Like}(x) | \text{Fit}(x)) \right)^F \\
\times \sum_{\text{Fash}(t)=1}^{5} \left[ P_{F_a}(\text{Like}(t) | \text{Fash}(t)) \right. \times P_{t,x}(\text{Fash}(t) | \overline{\text{Adv}}) \left. \right]\right\}
\]
Now we “open up” the last term in (16), $P_{l,x}(\text{Fash}(t) | \overline{\text{Adv}}(t))$, as follows

$$p_{l,x}(\text{Fash}_i(t) | \overline{\text{Adv}}(t)) = \sum_{Fash_{j-1}(t-1) = 1}^{5} \sum_{Fash_{j-1}(t-1) = 1}^{5} \ldots \sum_{Fash_{j-1}(t-1) = 1}^{5} P_{l,x}(\text{Fash}_i(t), \text{Fash}_i(t-1), \text{Fash}_j(t), \ldots, \text{Fash}_j(t-1) | \overline{\text{Adv}}(t))$$

and we approximate

$$P_{l,x}(\text{Fash}_i(t), \text{Fash}_i(t-1), \text{Fash}_j(t), \ldots, \text{Fash}_j(t-1) | \overline{\text{Adv}}(t))$$

$$\propto P_{l,x}(\text{Fash}_i(t), \text{Fash}_i(t-1) | \overline{\text{Adv}}(t))$$

$$\times \prod_{j=1}^{N_i} P_{l,x}(\text{Fash}_i(t), \text{Fash}_j(t-1) | \overline{\text{Adv}}(t-1))$$

$$= P_{l,x}(\text{Fash}_i(t) | \overline{\text{Adv}}(t), \text{Fash}_i(t-1)) \times P_{l,x}(\text{Fash}_i(t-1) | \overline{\text{Adv}}(t-1))$$

$$\times \prod_{j=1}^{N_i} \left[ P_{l,x}(\text{Fash}_i(t) | \text{Fash}_j(t-1)) \times P_{l,x}(\text{Fash}_j(t-1) | \overline{\text{Adv}}(t-1)) \right]$$

Resulting in

$$P_{l,x}(\text{Fash}_i(t) | \overline{\text{Adv}}(t))$$

$$= \frac{1}{Z_{\text{Fash}, \text{Fash}_i(t-1)}} \sum_{Fash_{j-1}(t-1) = 1}^{5} \left[ P_{l,x}(\text{Fash}_i(t) | \overline{\text{Adv}}(t), \text{Fash}_i(t-1)) \times P_{l,x}(\text{Fash}_i(t-1) | \overline{\text{Adv}}(t-1)) \right]$$

$$\times \prod_{j=1}^{N_i} \sum_{Fash_{j-1}(t-1) = 1}^{5} \left[ P_{l,x}(\text{Fash}_i(t) | \text{Fash}_j(t-1)) \times P_{l,x}(\text{Fash}_j(t-1) | \overline{\text{Adv}}(t-1)) \right]$$
Thus, the model (16) can be finally written as

\[
P_{i,x}(\text{Purchase}| \text{Trust, Fit, Price, Adv}) = P_{i,x}(\text{Purchase}| \text{Price}) \\
\times \sum_{\text{Like}(t)=1}^{5} \left\{ P_{i,x}(\text{Purchase}| \text{Like}(t)) \times \frac{1}{Z_{\text{Like}}} \left( P_{\text{Trust}}(\text{Like}_{i}(x)| \text{Trust}(x)) \right)^{T} \times \left( P_{\text{Fit}}(\text{Like}_{i}(x)| \text{Fit}(x)) \right)^{F} \right\} \\
\times \frac{1}{Z_{\text{Fash}}} \sum_{\text{Fash}(t)=1}^{5} \left\{ \left( P_{\text{Fash}}(\text{Like}_{i}(x)| \text{Fash}_{i}(x)) \right)^{Fa} \right\} \\
\times \prod_{j=1}^{N_{i}} \sum_{\text{Fash}_{j}(t-1)=1}^{4} \left\{ P_{i,j,x}(\text{Fash}_{i}(t)| \text{Fash}_{j}(t-1)) P_{j,x}(\text{Fash}_{j}(t-1)| \text{Adv}(t-1)) \right\}
\]
E. Homework 5 – Simulate this model

A spread sheet is provided.