Buying a Pair of Jeans

a. Write a file with all inputs for three(3) experts, agents 1,2,3.

Let us consider six(6) pair of jeans, i.e., $x=1,...,6$.

The inputs for each expert are: $Budget(i), Price(x), Trust(x), Fit(x)$, and initial state $\hat{S}_{Fash(i,x,t=0)}$. From the initial state we build the initial probability

$$P_i\left(Fash(i, x, t = 0)\right) = \begin{cases} 1 & Fash(i, x, t = 0) = \hat{Fash}(i, x, t = 0) \\ 0 & Fash(i, x, t = 0) \neq \hat{Fash}(i, x, t = 0) \end{cases}$$

where $\hat{Fash}(i, x, t = 0) = 0.25 * \hat{S}_{Fash(i,x,t=0)}$.

Also we need an advertising campaign for 10 periods, i.e., up to $t=10$, i.e., a set of values for

$$\overline{Adv}(x, t = 10) = \{(Adv(j, x, t')); j = 1, ..., N_i \text{ and } t' = 1, ..., t = 10\}$$

Finally we will need values for the parameters of the model(s):

$\rho_i, P_i, \lambda_i, \Lambda_i, \phi_i, F_i, \tau_i, T_i, \gamma_i, \Gamma_i, \Delta_{lx}, \alpha_{ij}, \Lambda_{ij}$.

Put all these data in the excel spread sheet provided with this homework, where your program can read from. We want to build a data base consistent with all students so that we can exchange spread sheets/data.

BuyingJeans.Hw5.xlsx
b. Write a subroutine to simulate the purchase probability or expected purchase value and plot its values over time, for $t=1,...,10$.

The expected purchase value is given by

$$
\langle Purchase(i, x, t) \rangle = \frac{1}{1 + e^{M_{i,price-Like}(Purchase(i, x, t) = 1)}}
$$

(11)

where

$$
M_{i,price-Like}(Purchase(i, x, t)) = M_{i,price}(Purchase(i, x, t)) + M_{i,Like}(Purchase(i, x, t))
$$

(9)

$$
= Purchase(i, x, t) \left[ \rho_i \left( \frac{price(x)}{Budget(i) - price(x)} \right)^{\nu_i} + \lambda_i \left( 1 - 0.25 \times S_{Like(i, x, t)} \right)^{\Delta_i} - 2 \right]
$$

Now, the parameters $\rho_i$ and $\lambda_i$ can be interpreted as weights for combining the two methods. The input is $Price(x), Like(i, x, t)$ and output is the expected value $\langle Purchase(i, x, t) \rangle$. If this expected value is larger than a threshold $T$, say $T=0.5$, than a purchase is made.

The input $Like(i, x, t)$ will need to be estimated from the subroutine in part c.

c. Write a subroutine to simulate the probability

$$
P_{i,x}(Like(i, x, t) | Fash(i, x, t), Trust(x), Fit(x)) = \frac{1}{Z_{Like}} e^{-M_{i,FTFA}(Like(i, x, t))}
$$

(19)

with the normalization constant

$$
Z_{Like} = \sum_{S_{Like(i, x, t)} = 0}^4 e^{-M_{i,FTFA}(Like(i, x, t) = 0.25 \times S_{Like(i, x, t)})}
$$

(20)

and

$$
M_{i,FTFA}(Like(i, x, t)) = M_{i,Fit}(Like(i, x, t)) + M_{i,Trust}(Like(i, x, t)) + M_{i,Fash}(Like(i, x, t))
$$

(21)

$$
= \phi_i \left( 0.25 \times S_{Like(i, x, t)} - Fit \right)^{\Phi_i} + \tau_i \left( 0.25 \times S_{Like(i, x, t)} - Trust \right)^{T_i}
$$

$$
+ \gamma_i \left( 0.25 \times S_{Like(i, x, t)} - S_{Fash(i, x, t)} \right)^{\Gamma_i}
$$

Thus, the coefficients $(\phi_i, \tau_i, \gamma_i)$ can be thought as the weights to combine the experts and one can think of using Boosting to estimate their values. We may then obtain the expected value.
Alternatively we estimate the optimal value
\[ \text{Like}(i, x, t) = 0.25 \times \hat{S}_{\text{Like}(i, x, t)} = \arg\min_{S_{\text{Like}(i, x, t)}} M_{i, FT Fa}(S_{\text{Like}(i, x, t)}) \]

and in this case one may not need to estimate \( Z_{i, \text{Like}} \). You can choose which one to use. The inputs are \( Fash(i, x, t), Trust(x), Fit(x) \) and one must have coefficients \( (\phi_i, \Phi_i, \tau_i, \tau_i, y_i, \Gamma_i) \).

The input \( Fash(i, x, t) \) is obtained from subroutine developed in part d.

d. Write a subroutine to simulate the probability

\[ P_i \left( Fash(i, x, t) \bigg| \overrightarrow{Adv}(x, t) \right) \]

Here \( \overrightarrow{Adv}(x, t) \) is a vector

\[ \overrightarrow{Adv}(x, t) = \{(Adv(j, x, t')); j = 1, ..., N_i \text{ and } t' = 1, ..., t\} \]

where \( Adv(j, x, t') \) is a parameter controlling the amount of advertising being absorbed by agent \( j \) about pair of jeans \( x \) at a moment in time \( t \).

We assume a state in fashion to be given at \( t=0 \), \( \overrightarrow{Fash}(i, x) = 0.25 \times \hat{S}_{Fash(i, x)} \), i.e.,

\[ P_i \left( Fash(i, x, t = 0) \bigg| \overrightarrow{Adv}(x, t = 0) \right) = \begin{cases} 1 & Fash(i, x, t = 0) = \overrightarrow{Fash}(i, x) \\ 0 & Fash(i, x, t = 0) \neq \overrightarrow{Fash}(i, x) \end{cases} \]
Use the formula

$$P_i \left( \text{Fash}(i,x,t) \bigg| \overline{\text{Adv}}(x,t) \right) =$$

$$= \frac{1}{Z_{\text{Fash}}} \left\{ \sum_{S_{\text{Fash}(i,x,t-1)}}^4 \left[ e^{-\overline{\text{Adv}}(i,x,t) | S_{\text{Fash}(i,x,t)-\min(S_{\text{Fash}(i,x,t-1)} + 1, 4) \rangle}^{\Delta_{i,x} + \alpha_{i} | S_{\text{Fash}(i,x,t)} - S_{\text{Fash}(i,x,t-1)} |^A_{i,i} } \right] \times P_i \left( \text{Fash}(i,x,t-1) \bigg| \overline{\text{Adv}}(x,t-1) \right) \right\}$$

$$\times \left\{ \prod_{j=1}^{N_i} \sum_{j \neq i}^4 \left[ e^{-\alpha_{i,j} | S_{\text{Fash}(i,x,t)} - S_{\text{Fash}(j,x,t-1)} |^A_{i,j} } \right] \times P_i \left( \text{Fash}(j,x,t-1) \bigg| \overline{\text{Adv}}(x,t-1) \right) \right\}$$

From this subroutine, you can iterate in time to estimate a sequence of probabilities $P_i \left( \text{Fash}(i,x,t) \bigg| \overline{\text{Adv}}(x,t) \right)$ over time and for each agent $i$ and each pair of jeans $x$.

We can then estimate the expected value

$$\langle \text{Fash}(i,x,t) \rangle = \sum_{S_{\text{Fash}(i,x,t)}=0}^4 \text{Fash}(i,x,t) \times P_i \left( \text{Fash}(i,x,t) \bigg| \overline{\text{Adv}}(x,t) \right)$$

$$= \sum_{S_{\text{Fash}(i,x,t)}=0}^4 0.25 \times S_{\text{Fash}(i,x,t)} \times P_i \left( S_{\text{Fash}(i,x,t)} \bigg| \overline{\text{Adv}}(x,t) \right)$$

as output. Alternatively, select the state $\hat{S}_{\text{Fash}(i,x,t)} \in \{0,1,2,3,4\}$ that maximizes the probability

$$\hat{S}_{\text{Fash}(i,x,t)} = \arg \max_{S_{\text{Fash}(i,x,t)}} P_i \left( \text{Fash}(i,x,t) = 0.25 * S_{\text{Fash}(i,x,t)} \bigg| \overline{\text{Adv}}(x,t) \right)$$

Then, use $\overline{\text{Fash}}(i,x,t) = 0.25 * \hat{S}_{\text{Fash}(i,x,t)}$ as input to the subroutine of part c.