Graphical Models for Buying a Pair of Jeans

A. Statement of the Variables

An agent wants to buy a pair of jeans and there are \( n \) different stores to buy. Say for simplicity, there is one pair of jeans per brand, so jeans and brands are being represented by the same variable. How to choose? Which variables play a role in making such decision and how a decision is made? Let us refer to the variable \( x = 1, \ldots, n \) for each pair of jeans. Say each agent is represented by the variable \( i \). Say the agent must decide if will purchase or not the pair of Jeans, i.e., we need a binary variable \( \text{Purchase}(i, x) = 0, 1 \) to represent such decision. There is also a variable \( \text{Fash}(i, x) \) that represents how much agent \( i \) thinks brand \( x \) is fashionable. Fashionable is the result of many different considerations by agent \( i \) and we need to model it.

\**Wallet.** How much money to spend and the price of the pair of jeans in each store? So say the cost is \( \text{Price}(x) \) and the budget to buy a pair of jeans per agent is \( \text{Budget}(i) \). Of course, \( \text{Price}(x) \leq \text{Budget}(i) \), for agent \( i \) to be able to buy pair of jeans \( x \). Also, it is clear that the lower the price the more agent \( i \) is willing to buy it (sometimes agents may prefer to buy if it is more expensive).

\**Like.** Agent \( i \) must like a pair of jeans \( x \) to want to buy it. “Like” may involve Fit, Trust on a brand and how Fashionable is a pair of Jeans. “Like” maybe modeled as a random variable in the range \( [0, 1] \) or if we need to discretize for computational purposes, we may assign to it five states states, say “don’t Like”, “maybe Like”, “Like”, “Like a lot”, “love it” so using some map such as \( S_{\text{Like}}(i, x) \in \{0, 1, 2, 3, 4\} \rightarrow \text{Like}(i, x) = 0.2 \times S_{\text{Like}}(i, x) = 0.0, 0.2, 0.4, 0.6, 0.8 \).

\**Fit.** Agent \( i \) must think the pair of Jeans fit well. This is a somewhat subjective perception, but one that agent \( i \) knows how evaluate. Let us refer to \( \text{Fit}(i, x) \) as a variable in \( [0, 1] \), where \( \text{Fit}=1 \) means perfect fit and 0 means no chance of fitting.

\**Trust.** Agent \( i \) must have a sense to trust or not a jeans \( x \). Maybe agent \( i \) bought in the past a jeans \( x \) from the same brand. Here we are assuming that \( x \) is brand as well as the unit. Maybe agent \( i \) heard that a particular brand does not last long or the other way around. So we may consider a variable \( \text{Trust}(i, x) \in [0, 1] \), to represent how much the customer believes the brand of \( x \) is to be trusted, it is a good quality Jeans. This reflect the agent \( i \) perception of the history of the brand \( x \).

\**Fashion.** Agent \( i \) have a sense of fashion. Fashion may include the color of the pair of Jeans \( x \), and many other important details, such as (i) “Washing” types, (ii) “Waist-height” such as low, regular, high, (iii) the cut/style (boot, straight, flair, cigarette, skinny), (iv) color (light, grey, blue, …), (v) ragged, (vi) pocket styles. Fashion may also be acquired by other factors, such as advertising and other agents around agent \( i \). This sense of fashion influences agent \( i \) to prefer one pair of jeans over the other. Let us
refer to the variable \( Fash(i,x) \) which is the fashion perception of agent \( i \) about jeans \( x \). So Fashion, similarly to \( Like(i,x) \), maybe modeled as a random variable in the range \([0,1]\) or if we need to discretize for computational purposes, we may assign to it five states states, say “not fashionable”, “maybe fashionable” “fashionable”, “very fashionable”, “fantastic” so these states must map to \([0,1]\) values in some way, such as \( S_{Fash}(i,x) \in \{0,1,2,3,4\} \rightarrow Fash(i,x)=0.2 \times S_{Fash}(i,x) = 0,0.2,0.4,0.6,0.8 \).

**Advertising.** This measures how much advertising is hitting a customer. Advertising is capable of making a pair of jeans \( x \) to become more fashionable and/or more trustable. Advertising maybe able to make an item that is otherwise as \( S_{Fash}(i,x) = 1 \) to become as \( S_{Fash}(i,x) = 2 \), for example. \( Adv(i,x) \) maybe thought as an external force working on the agent to change fashion states.

**Social Behavior.** Every agent has its community for a given subject. The subject here is buying a pair of jeans. Agent \( i \) may have its close friends, a set \( N_i \) of agents to whom to consult. It may also have some other friends, not so close, to consult about it. They all contribute for agent \( i \) to have a fashion sense.

### B. Graphical Model

#### B1. One Agent

**Probabilities, States and Variables**

- \( Purchase_i_t (A,B,...X) \)
- \( Like_i_t (A,B,...X) \)
- \( Advertise_i_t (A,B,...X) \)
- \( Fash_i_t (A,B,...X) \)
- \( Fit_i (A,B,...X) \)
- \( Trust_i (A,B,...X) \)
- \( Budget_i \)
- \( Price(A,B,...X) \)

**Diagram**

- **Agent\(_i\)**
- **Brand A**, **Brand B**, **Brand X**
- **Like**, **Fit**, **Trust**, **Price & Budget**
- **Purchase**
- **Advertise**
- **Time**
B2. Many Connected Agents

Probabilities, States and Variables

Purchase_i_t (A,B,...X)

Like_i_t (A,B,...X)

Advertise_i_t (A,B,...X)

Fash_i_t (A,B,...X)

Fit_i (A,B,...X)

Trust_i (A,B,...X)

Budget_i

Price(A,B,...X)
C. Basic Probability

**Probabilities over discrete variables.** Say we have a random variable $S$ that can take discrete label states, $\mathcal{L} = \{0,1,2,3,4\}$, i.e., $S \in \{0,1,2,3,4\}$. Say there is another random variable $R$ taking values on the same label set $\mathcal{L}$, $R \in \{0,1,2,3,4\}$ and you have access to the distributions $P(S|R)$, $P(R)$ and wish to find the distribution $P(S)$. The following probability relations are known to be valid

$$P(S) = \sum_{R \in \mathcal{L}} P(S,R) = \sum_{R \in \mathcal{L}} P(S|R) P(R) \quad (C.1)$$

There are $|\mathcal{L}|$ computations (sums), where $|\mathcal{L}|$ is the size of the set $\mathcal{L}$, in our example $\mathcal{L} = \{0,1,2,3,4\}$ is $|\mathcal{L}| = 5$. We can extend to multiple variables, say $N$ variables $R_1, R_2, ..., R_N$, each one taking values on the same label set $\mathcal{L}$. Then, formula (1) becomes

$$P(S) = \sum_{R_1 \in \mathcal{L}} \sum_{R_2 \in \mathcal{L}} \cdots \sum_{R_N \in \mathcal{L}} P(S,R_1,R_2,...,R_N)$$

$$= \sum_{R_1 \in \mathcal{L}} \sum_{R_2 \in \mathcal{L}} \cdots \sum_{R_N \in \mathcal{L}} P(S|R_1,R_2,...,R_N) P(R_1,R_2,...,R_N) \quad (C.2)$$

These sums represent $|\mathcal{L}|^N$ computations (sums). For the special case where

$$P(S|R_1,R_2,...,R_N) = P(S|R_1)P(S|R_2) \cdots P(S|R_N) \quad \text{and} \quad P(R_1,R_2,...,R_N) = P(R_1)P(R_2) \cdots P(R_N) \quad (C.3)$$

It is clear that formula (2) becomes

$$P(S) = \sum_{R_1 \in \mathcal{L}} P(S|R_1)P(R_1) \times \sum_{R_2 \in \mathcal{L}} P(S|R_2)P(R_2) \times \cdots \times \sum_{R_N \in \mathcal{L}} P(S|R_N)P(R_N) \quad (C.4)$$

There are $N |\mathcal{L}|$ sum computations and there are $N - 1$ product computations. It is clear that for large values of $N$, $N (|\mathcal{L}| + 1) - 1 \ll |\mathcal{L}|^N$, and thus, when formula (C.3) applies, the problem becomes computationally much more feasible than when must apply formula (C.2) to compute $P(S)$.

**Probabilities over time.** There is a case where these variables evolve over time. For example, say $S$ is $S(t)$. We may have the so called Markov case, in time, where the dependency of $S(t)$ is only to $S(t-1)$, and not previous times. We also may have many agents, indexed by $i = 1, ..., N$, and so the variable $S_i(t)$ refers to the possible states of agent $i$ at time $t$. Analogous to formula (C.2), replacing the variables $R$ by $S(t-1)$ we can write
Similar to formula (C.3) we can sometimes have the simplification

$$P(S_i(t)|S_1(t-1), S_2(t-1), ..., S_N(t-1)) = P(S_i(t)|S_1(t-1))P(S_i(t)|S_2(t-1)) ... P(S_i(t)|S_N(t-1))$$

(C.6)

and

$$P(S_1(t-1), S_2(t-1), ..., S_N(t-1)) = P(S_1(t-1))P(S_2(t-1)) ... P(S_N(t-1))$$

(C.7)

and in this case formula (C.5) becomes

$$P(S_i(t)) = \left( \sum_{S_1(t-1) \in \mathcal{L}} [P(S_i(t)|S_1(t-1)) P(S_1(t-1))] \right) \times \left( \sum_{S_2(t-1) \in \mathcal{L}} [P(S_i(t)|S_2(t-1)) P(S_2(t-1))] \right) \times ... \times \left( \sum_{S_N(t-1) \in \mathcal{L}} [P(S_i(t)|S_N(t-1)) P(S_N(t-1))] \right)$$

(C.8)

We have \(N|\mathcal{L}\) sum computations and \(N - 1\) multiplications to evaluate the probability, i.e., it is a linear algorithm on the number of variables and linear on the number of states.

This process allows to compute \(P(S_i(t))\) for all agents \(i\), given the same probabilities evaluated at time \(t-1\). We need to also have the transition probabilities \(P\left(S_i(t)|S_j(t-1)\right)\) for each pair of agent \(i\) and agent \(j\).

**Estimation.** Estimation is the “science” of estimating variables from probability distributions. Given \(P(S_i(t))\) what is the best estimate for \(S_i(t)\)? One possible estimator is to choose \(\hat{S}_i(t)\) that maximizes the probability \(P(S_i(t))\), i.e., the MAP estimator (maximum a posterior probability) is defined as

$$\hat{S}_i(t) = \text{argmax}_{S_i(t)} P(S_i(t))$$

(C.9)

Another one is the MPM estimator (maximal posterior marginal estimator) which is defined as

$$\langle S_i(t) \rangle = \sum_{S_i(t) \in \mathcal{L}} S_i(t) \times P(S_i(t))$$

(C.10)
The MPM minimizes the variance, i.e., \( \langle S_i(t) \rangle \) minimizes \( \text{Var}(\overline{S_i(t)}) \) where

\[
\text{Var}(\overline{S_i(t)}) = \sum_{S_i(t) \in L} (S_i(t) - \overline{S_i(t)})^2 \times P(S_i(t)) \quad \rightarrow \quad \frac{\partial \text{Var}(\overline{S_i(t)})}{\partial \overline{S_i(t)}} \bigg|_{\langle S_i(t) \rangle} = 0 \quad \rightarrow \quad \langle S_i(t) \rangle = \sum_{S_i(t) \in L} S_i(t) \times P(S_i(t)) \quad (C.11)
\]

and it is often thought to be a better estimator.

**D. Purchase Modeling**

**. Wallet and Purchase.** Let us say that the lower the price of the pair of jeans x the more likely agent \( i \) is to buy it. So a formula for this is

\[
P_i(Purchase(i, x, t)|Price(x)) = \frac{1}{Z} e^{-Purchase(i,x)} \left[ \rho_i \left( \frac{Price(x)}{Budget(i)-Price(x)} \right)^{P_i} \right]^{P_i} \quad (1)
\]

The parameters \( \rho_i, P_i, T \) describe the behavior of agents when buying jeans. \( Z \) is a normalization constant so that \( 1 = P_i(0|Price(x)) + P_i(1|Price(x)) \), i.e.,

\[
Z = 1 + e^{-\left[ \rho_i \left( \frac{Price(x)}{Budget(i)-Price(x)} \right)^{P_i} \right]}^{P_i} - T_P
\]

To understand the parameter \( T \), we can observe that if \( Price(x) = 0 \), the probability of purchase becomes

\[
P_i(1|Price(x)) = \frac{e^{TP_i}}{1 + e^{TP_i}}
\]

And the larger is \( T_P \) the more likely agent \( i \) will purchase.

The parameter \( P_i \) controls how “non-linear” is the behavior of agent \( i \) with respect to \( Price(x) \). Both agents are more likely to buy if \( Price(x) \) decreases. For any given \( Price(x) \) the exact behavior depends if \( P_i > 1 \) or \( P_i < 1 \). The following table compares the behavior of agents to purchase, given a \( Price(x) \), depending which regime of parameters they have

<table>
<thead>
<tr>
<th>( \frac{Price(x)}{Budget(i)-Price(x)} )</th>
<th>( P_i &lt; 1 )</th>
<th>( P_i &gt; 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small prices</td>
<td>Less likely to buy</td>
<td>More likely to buy</td>
</tr>
<tr>
<td>Large prices</td>
<td>More likely to buy</td>
<td>Less likely to buy</td>
</tr>
</tbody>
</table>

So one may say that \( P_i > 1 \) characterizes an agent that likes to buy cheap clothes and \( P_i < 1 \) is the behavior of an agent that likes to buy expensive clothes. This statement should be understood to be
relative to each other, since both prefer to pay less than to pay more. One may look at the extremes to better understand the statement. If $P_1 \to \infty \geq 1$ agent will only want to buy clothes that cost nothing ($\text{Price}(x) = 0$) while in the other extreme, if $P_1 = 0 < 1$, agent will buy any pair of jeans regardless of its price.

The parameter $\rho_1$ scales linearly this behavior and its role will become more clear on the next sessions when combining different experts that contribute for purchase.

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Note that even though purchase interest does vary over time, the effect due to price does not vary over time. Other factors varying over time will cause purchase interest to vary over time.

Deterministic View. We can also think this as a cost model or a “method” like we did for Checkers. More precisely,

$$M_{i,\text{price}}(\text{Purchase}(i, x, t)) = \text{Purchase}(i, x, t)\left(\rho_1 \left(\frac{\text{Price}(x)}{\text{Budget}(i)-\text{Price}(x)}\right)^{P_1} - T_p\right)$$

(2)

where $\text{Purchase}(i, x) = 0, 1$ is a binary variable. Agent $i$ goal is to minimize the cost model, i.e., to purchase or not the pair of jeans $x$ in order to minimize the cost, given the price and budget and the parameters $\rho_1, P_1$.

Likeable and Purchase. Likeable makes agent $i$ more likely to buy $x$. We will assume that over time, through advertising and social interactions, like will change, so it will be modeled using time as a variable, using the formula

$$P_1(\text{Purchase}(i, x, t) = 1|\text{Like}(i, x, t)) = \frac{1}{Z_L} e^{-\text{Purchase}(i, x, t) \left[\lambda_i \left[1 - \text{Like}(i, x, t)\right]^\Lambda_1 - T_L\right]}$$

(5)

and the coefficients $(\lambda_i, \Lambda_1, T_L, Z_L)$ must be calculated. $\text{Like}(i, x, t)$ is a variable in the range $[0,1]$. We can also model it in terms of states $S_{\text{Like}(i,x,t)} = \{0,1,2,3,4\}$ as

$$P_1(\text{Purchase}(i, x, t) = 1|\text{Like}(i, x, t)) = \frac{1}{Z_L} e^{-\text{Purchase}(i, x, t) \left[\lambda_i \left[1 - 0.2 \cdot S_{\text{Like}(i,x,t)}\right]^\Lambda_1 - T_L\right]}$$

(6)

Note that the parameter $\Lambda_1$ can be understood similarly to the previous session, where the table was created. In this case, the term $\left[1 - 0.2 \cdot S_{\text{Like}(i,x,t)}\right]$ is always $\left[1 - 0.2 \cdot S_{\text{Like}(i,x,t)}\right] \leq 1$. So, for $\Lambda_1 < 1$ the effect of liking more a product will not affect as much the decision to purchase compared to $\Lambda_1 > 1$.

Deterministic view. We can also think this as a cost model, or method, using the state variable $S_{\text{Like}(x)} = 0, 1, 2, 3, 4$. 
Combining Likeable and Price to make a Purchase. One can combine these two methods (like and price) by multiplying the probabilities, i.e., to model it as

\[
M_{i,\text{Like}}(\text{Purchase}(i, x, t)) = \text{Purchase}(i, x, t) \left[ \lambda_i \left| 1 - 0.2 \times S_{\text{Like}(i, x, t)} \right|^{\Lambda_i} - T_L \right]
\]  

(7)

Alternative way of interpreting: Boosting or combining methods.

By modeling each probability via a cost model, or method, we would add the cost models to get the final one. More precisely

\[
P_i(\text{Purchase}(i, x, t)|\text{Price}(x), \text{Like}(i, x, t)) = \frac{1}{Z_{\text{pur}}} \left[ P_i(\text{Purchase}(i, x, t)|\text{Price}(x)) \times P_i(\text{Purchase}(i, x, t)|\text{Like}(i, x, t)) \right]
\]

where

\[
Z_{\text{pur}} = P_i(\text{Purchase}(i, x, t) = 1|\text{Price}(x)) \times P_i(\text{Purchase}(i, x, t) = 1|\text{Like}(i, x, t)) + P_i(\text{Purchase}(i, x, t) = 0|\text{Price}(x)) \times P_i(\text{Purchase}(i, x, t) = 0|\text{Like}(i, x, t)).
\]

Expected values of Purchase.

This probability yields an expected value
\[ \langle \text{Purchase}(i, x, t) \rangle \]

\[ = \sum_{\text{purchase}(i,x,t)=0}^{1} \text{Purchase}(i,x,t) \times P_i(\text{Purchase}(i,x,t) | \text{Price}(x), \text{Like}(i,x,t)) \]

\[ = 1 \times \frac{e^{-M_{i,\text{price-Like}}(\text{Purchase}(i,x,t)=1)}}{1 + e^{-M_{i,\text{price-Like}}(\text{Purchase}(i,x,t)=1)}} \]

\[ = \frac{1}{1 + e^{M_{i,\text{price-Like}}(\text{Purchase}(i,x,t)=1)}} \quad (11) \]

**Like: Fit, Trust, Fashion.** Which elements makes agent \( i \) more likely to think that \( x \) is Likeable?

We assume we have the independence

\[ P_{Lx}(\text{Like}(i,x,t) | \text{Fash}(i,x,t), \text{Trust}(x), \text{Fit}(x)) \]

\[ = \frac{1}{Z_{\text{Like}}} P_{\text{Trust}}(\text{Like}(i,x,t) | \text{Trust}(x)) P_{\text{Fit}}(\text{Like}(i,x,t) | \text{Fit}(x)) P_{\text{Fash}}(\text{Like}(i,x,t) | \text{Fash}(i,x,t)) \quad (12) \]

If we think each probability as being generated by a method, we write

\[ P_{\text{Fash}}(\text{Like}(i,x,t) | \text{Fash}(i,x,t)) = \frac{1}{Z_{\text{Fa}}} e^{-M_{i,\text{Fash}}(\text{Like}(i,x,t))} \quad (13) \]

where, for example,

\[ M_{i,\text{Fash}}(\text{Like}(i,x,t)) = \gamma_i \ 0.25^{T_i} \left| S_{\text{Like}(i,x,t)} - S_{\text{Fash}(i,x,t)} \right|^{T_i} \quad (14) \]

where we wrote \( \text{Fash}(i,x) \) in terms of the discrete states \( S_{\text{Fash}(i,x)} \) and \( \text{Like}(i,x,t) \) in terms of the discrete states \( S_{\text{Like}(i,x,t)} \).

Similarly, let us use a model for “Like” given Fit and “Like” given Trust, remembering the better is the \( \text{Fit} \) and the better is \( \text{Trust} \) should imply the better is \( \text{Like} \). For example using the models

\[ P_{\text{Trust}}(\text{Like}(i,x,t) | \text{Trust}) = \frac{1}{Z_T} e^{-M_{i,\text{Trust}}(\text{Like}(i,x,t))} \quad (15) \]

\[ M_{i,\text{Trust}}(\text{Like}(i,x,t)) = \tau_i \left| 0.25 \ast S_{\text{Like}(i,x,t)} - \text{Trust} \right|^{T_i} \quad (16) \]

and

\[ P_{\text{Fit}}(\text{Like}(i,x,t) | \text{Fit}) = \frac{1}{Z_T} e^{-M_{i,\text{Fit}}(\text{Like}(i,x,t))} \quad (17) \]

\[ M_{i,\text{Fit}}(\text{Like}(i,x,t)) = \phi_i \left| 0.25 \ast S_{\text{Like}(i,x,t)} - \text{Fit} \right|^{\Phi_i} \quad (18) \]
Combining the three methods

Our goal is to write

\[ P_{Lx}(\text{Like}(i, x, t) \mid Fash(i, x, t), \text{Trust}(x), \text{Fit}(x)) = \frac{1}{Z_{\text{Like}}} e^{-M_{i,FTFa}(\text{Like}(i, x, t))} \]  

(19)

with the normalization constant

\[ Z_{\text{Like}} = \sum_{S_{\text{Like}(i, x, t)} = 0}^{4} e^{M_{i,FTFa}(\text{Like}(i, x, t) = 0.25 \cdot S_{\text{Like}(i, x, t)})} \]  

(20)

With the independence assumption in (10) and with the methods (12), (14), (16) we obtain

\[ M_{i,FTFa}(\text{Like}(i, x, t)) = M_{i,Fit}(\text{Like}(i, x, t)) + M_{i,Trust}(\text{Like}(i, x)) + M_{i,Fash}(\text{Like}(i, x)) \]  

(21)

\[ = \Phi_i \cdot 0.25 \cdot S_{\text{Like}(i, x, t)} - \text{Fit} |^{\Phi_i} + \tau_i \cdot 0.25 \cdot S_{\text{Like}(i, x, t)} - \text{Trust} |^{\tau_i} \]

\[ + \gamma_i \cdot 0.25 |^{\gamma_i} S_{\text{Like}(i, x, t)} - S_{\text{Fash}(i, x, t)} |^{\gamma_i} \]

Thus the coefficients \((\Phi_i, \tau_i, \gamma_i)\) can be thought as the weights to combine the experts and one can think of using Boosting to estimate their values. We can then estimate the expected value

\[ \langle \text{Like}(i, x, t) \rangle = \sum_{S_{\text{Like}(i, x, t)} = 0}^{4} \text{Like}(i, x, t) \times \frac{1}{Z_{\text{Like}}} e^{-M_{i,FTFa}(\text{Like}(i, x, t))} \]  

(22)

\[ = \sum_{S_{\text{Like}(i, x, t)} = 0}^{4} 0.25 \times S_{\text{Like}(i, x, t)} \times \frac{1}{Z_{\text{Like}}} e^{-M_{i,FTFa}(S_{\text{Like}(i, x, t)})} \]

Alternatively, one may simply estimate the value \(\overline{\text{Like}}(i, x, t) = 0.25 \cdot S_{\text{Like}(i, x, t)}\) that minimizes \(M_{i,FTFa}(\text{Like}(i, x, t) = 0.25 \cdot S_{\text{Like}(i, x, t)})\), i.e.,

\[ \overline{\text{Like}}(i, x, t) = 0.25 \cdot S_{\text{Like}(i, x, t)} \]

\[ = \arg \min_{S_{\text{Like}(i, x, t)}} M_{i,FTFa}(\text{Like}(i, x, t) = 0.25 \cdot S_{\text{Like}(i, x, t)}) \]  

(23)

and one may not need to estimate \(Z_{\text{Like}}\).

Now we introduce Advertising and Social behavior, and how they will impact the changes of agent \(i\) states over time. Our goal is to generate a model for \(P_i \left( Fash(i, x, t) \mid \overline{\text{Adv}}(x, t) \right)\) that summarizes both, the social influence of other agents as well as advertising campaigns. Here \(\overline{\text{Adv}}(x, t)\) is a vector
where \( \text{Adv}(j, x, t') \) is a parameter controlling the amount of advertising being absorbed by agent \( j \) about a pair of jeans \( x \) at a moment in time \( t \).

Advertising acts as a force, to increase fashionable strength to brand \( x \). The following formula may characterize such operation via the probability model

\[
P_i(Fash(i, x, t) | \text{Adv}(i, x, t), Fash(i, x, t-1)) = \frac{1}{Z_{\text{Adv}}} e^{-\text{Adv}(i, x, t) | S_{\text{Fash}(i,x,t)} - \min(S_{\text{Fash}(i,x,t-1)} + 1, 4)) | A_{ix}} e^{-a_{ii} | S_{\text{Fash}(i,x,t)} - S_{\text{Fash}(i,x,t-1)} | A_{ii}} = \frac{1}{Z_{\text{Adv}}} e^{-\text{Adv}(i, x, t) | S_{\text{Fash}(i,x,t)} - \min(S_{\text{Fash}(i,x,t-1)} + 1, 4)) | A_{ix} + a_{ii} | S_{\text{Fash}(i,x,t)} - S_{\text{Fash}(i,x,t-1)} | A_{ii}} (25)
\]

or thinks as a method

\[
M_{i, \text{Adv}}(Fash(i, x, t)) = \text{Adv}(i, x, t) | S_{\text{Fash}(i,x,t)} - \min(S_{\text{Fash}(i,x,t-1)} + 1, 4)) | A_{ix} + a_{ii} | S_{\text{Fash}(i,x,t)} - S_{\text{Fash}(i,x,t-1)} | A_{ii} (26)
\]

where the lower the value of \( M_{i, \text{Adv}}(Fash(i, x, t)) \) the better. It suggests that the more advertising the more likely it is for the state \( S_{\text{Fash}(i,x,t)} \) to increase one unit, unless it is already at the highest state, \( S_{\text{Fash}(i,x,t)} = 4 \). Note that the advertising term itself, the first term, suggests that agent \( i \) is as likely to increase two states as it is to stay at the same state (except at the upper and lower bound states). Thus, the other term acts as an “inertia” term that causes the agent \( i \) to stay at the same state as it is on the previous time. This term may also be interpreted as part of a social behavior. Advertising is unlikely to have a negative effect of decreasing one state (it is the same strength as increasing three states in Fashionable, up to the upper and lower bounds states). The larger is \( \text{Adv}(i, x, t) \) the more these effects. The advertising parameter \( \Delta_{ix} \) and the social inertia behavior parameters \( a_{ii}, A_{ii} \) must be calculated.

BuyingJeans.Hw5.xlsx

Social Behavior.

Strong Ties. A close community of agent \( i \) is represented by \( j \in N_i \) and each \( j \) push each member to share their Fashion values. When \( j = 1 \) this model reflects an inertia of how agent \( i \) tends to stay put on its own state. This may be described by
or method

\[ M_{i,j, \text{Social}}(\text{Fash}(i,x,t)) = \alpha_{ij} | S_{\text{Fash}(i,x,t)} - S_{\text{Fash}(j,x,t-1)} |^{A_{ij}} \]  

Note that here we may be considering how agent \( j \) “thinks” jeans \( x \) will “look” at agent \( i \), and not on itself. For example, “fit” is factored on how agent \( j \) evaluates \( \text{Fash}(j,x,t) \), so it must be using the probability of fitting of jeans \( x \) to agent \( i \). While both close friends influence each other to get to the same fashion opinion on Jeans \( x \), their influence does not have to be symmetric, i.e., \( \alpha_{ij} \neq \alpha_{ji} \). The case \( j=i \) is interpreted as how much agent \( i \) does not like to change its mind. It is the same term used in the advertising model. In order to normalize (25), we must have

\[ ZS_{ij} = \sum_{S_{\text{Fash}(i,x,t)}=0}^{4} e^{-\alpha_{ij}} | S_{\text{Fash}(i,x,t)} - S_{\text{Fash}(j,x,t-1)} |^{A_{ij}} \]  

Combining all agents \( j = 1, ..., N_i \), \( j \neq i \) and using equation (24) we get

\[ P_{i,x}(\text{Fash}(i,x,t)|\text{Fash}(j = 1, x, t), ..., \text{Fash}(j = N_i, x, t)) = \frac{1}{Z_f} \prod_{j=1}^{N_i} P_{i,j,x}(\text{Fash}(i,x,t)|\text{Fash}(j = 1,x,t)) = \frac{1}{Z_{fs_i}} e^{-\sum_{j=1}^{N_i} \alpha_{ij} | S_{\text{Fash}(i,x,t)} - S_{\text{Fash}(j,x,t-1)} |^{A_{ij}}} \]

with the normalization

\[ Z_{fs_i} = Z_f \prod_{j=1}^{N_i} ZS_{ij} = \sum_{S_{\text{Fash}(i,x,t)}=0}^{4} e^{-\sum_{j=1}^{N_i} \alpha_{ij} | S_{\text{Fash}(i,x,t)} - S_{\text{Fash}(j,x,t-1)} |^{A_{ij}} \]  

and the equivalent combination of methods gives

\[ M_{i, \text{Social}}(\text{Fash}(i,x,t)) = \sum_{j=1}^{N_i} \alpha_{ij} | S_{\text{Fash}(i,x,t)} - S_{\text{Fash}(j,x,t-1)} |^{A_{ij}} \]  

Combining Advertising and Agents Social Behavior from top down.

Our goal is to derive \( P_{t}(\text{Fash}(i,x,t)|\text{Adv}(x,t)) \). We model it as follows.
These are $5^{N_i}$ computations and can be too many. We use as reference section C. In order to simplify we will break these probability in terms of each variable alone where sums can be done independently.

\[
P_i \left( Fash(i, x, t) \, | \, Adv(x, t) \right) = \sum_{S_{Fash(i,x,t-1)=0}} \sum_{S_{Fash(N_i,x,t-1)=0}} P_i \left( Fash(i, x, t), Fash(j = 1, xt - 1), ..., Fash(j = N_i, x, t - 1) \, | \, Adv(x, t) \right)
\]

Say that we do have a probability model, at time $t-1$, $P_i \left( Fash(i, x, t-1) \, | \, Adv(x, t-1) \right)$, for all agents $i$. Then, using formulae C.5, C.6, C.7 we can derive

\[
P_i \left( Fash(i, x, t), Fash(j = 1, xt - 1), ..., Fash(j = N_i, x, t - 1) \, | \, Adv(x, t) \right) = P_i \left( Fash(i, x, t) \, | \, Adv(i, x, t), Fash(j = 1, xt - 1), ..., Fash(j = N_i, x, t - 1) \right) \\
\times P \left( Fash(j = 1, x, t - 1), ..., Fash(j = N_i, x, t - 1) \, | \, Adv(x, t - 1) \right)
\]

Breaking

\[
P_i \left( Fash(i, x, t) \, | \, Adv(i, x, t), Fash(j = 1, x, t - 1), ..., Fash(j = N_i, x, t - 1) \right) = \frac{1}{Z_1} P_i \left( Fash(i, x, t) \, | \, Adv(i, x, t), Fash(i, x, t - 1) \right) \times \left( \prod_{j=1, j\neq i}^{N_i} P_i \left( Fash(i, x, t) \, | \, Fash(j = 1, x, t - 1) \right) \right)
\]

and breaking

\[
P \left( Fash(j = 1, x, t - 1), ..., Fash(j = N_i, x, t - 1) \, | \, Adv(x, t - 1) \right) \\
= \frac{1}{Z_2} P_i \left( Fash(i, x, t - 1) \, | \, Adv(x, t - 1) \right) \times \prod_{j=1, j\neq i}^{N_i} P_j \left( Fash(j, x, t - 1) \, | \, Adv(x, t - 1) \right)
\]
We can then rewrite (31) as

\[
P_i \left( Fash(i, x, t), Fash(j = 1, x t - 1), ..., Fash(j = N_i, x, t - 1) \middle| \overline{Adv}(x, t) \right)
\]

\[
= \frac{1}{Z1 \times Z2} \sum_{S_{Fash(1, x t - 1) = 0}}^{4} \cdots \sum_{S_{Fash(N_i, x t - 1) = 0}}^{4} \left\{ P_i \left( Fash(i, x, t) \middle| Adv(i, x, t), Fash(i, x, t - 1) \right) \right. \\
\left. \times P_i \left( Fash(i, x, t - 1) \middle| \overline{Adv}(x, t - 1) \right) \right. \\
\left. \times \prod_{j=1 \atop j \neq i}^{N_i} \left[ P_i \left( Fash(i, x, t) \middle| Fash(j = 1, x, t - 1) \right) \times P_j \left( Fash(j, x, t - 1) \middle| \overline{Adv}(x, t - 1) \right) \right] \right\}
\]

and thus, we can rewrite (30) as

\[
P_i \left( Fash(i, x, t) \middle| \overline{Adv}(x, t) \right)
\]

\[
= \frac{1}{Z1 \times Z2} \sum_{S_{Fash(1, x t - 1) = 0}}^{4} \cdots \sum_{S_{Fash(N_i, x t - 1) = 0}}^{4} \left\{ P_i \left( Fash(i, x, t) \middle| Adv(i, x, t), Fash(i, x, t - 1) \right) \right. \\
\left. \times P_i \left( Fash(i, x, t - 1) \middle| \overline{Adv}(x, t - 1) \right) \right. \\
\left. \times \prod_{j=1 \atop j \neq i}^{N_i} \left[ P_i \left( Fash(i, x, t) \middle| Fash(j = 1, x, t - 1) \right) \times P_j \left( Fash(j, x, t - 1) \middle| \overline{Adv}(x, t - 1) \right) \right] \right\}
\]

which can also be written (by “pushing” the sums inside the products) as

\[
P_i \left( Fash(i, x, t) \middle| \overline{Adv}(x, t) \right)
\]

\[
= \left( \sum_{S_{Fash(1, x t - 1) = 0}}^{4} \left[ P_i \left( Fash(i, x, t) \middle| Adv(i, x, t), Fash(i, x, t - 1) \right) \times P_i \left( Fash(i, x, t - 1) \middle| \overline{Adv}(x, t - 1) \right) \right] \right) \\
\times \left( \prod_{j=1 \atop j \neq i}^{N_i} \sum_{S_{Fash(N_i, x t - 1) = 0}}^{4} \left[ P_i \left( Fash(i, x, t) \middle| Fash(j = 1, x, t - 1) \right) \times P_j \left( Fash(j, x, t - 1) \middle| \overline{Adv}(x, t - 1) \right) \right] \right)
\]

So we manage to reduce the computation of \( P_i \left( Fash(i, x, t) \middle| \overline{Adv}(x, t) \right) \) to 5x2x\( N_i \) computations and generate a recursive process over time. This recursive process evolves \( P_i \left( Fash(i, x, t) \middle| \overline{Adv}(x, t) \right) \) over time. We can now write (33) with its details as
Note that (34) is computationally "easy" to perform since we can perform the sum separately for each agent and there is only one normalization needed at the end, namely $Z_{Fash}$. We can estimate the expected value of $Fash(i, x, t)$ as

$$\langle Fash(i, x, t) \rangle = \sum_{S_{Fash(i,x,t)}=0}^4 Fash(i, x, t) \times P_l \left( Fash(i, x, t) \left| \overline{Adv}(x, t) \right. \right) \quad (38)$$

$$= \sum_{S_{Fash(i,x,t)}=0}^4 0.2 \times S_{Fash(i,x,t)} \times P_l \left( S_{Fash(i,x,t)} \left| \overline{Adv}(x, t) \right. \right)$$

**Weak Ties.** An acquaintance or expert in Fashion may influence agent $i$ in making his choices. This may be modeled as a “weak” tie. Let us refer to it as expert or agent $k$ not in $N_i$.

$$P_{ik}(Fash(i, x, t) | Fash(k, x, t - 1)) = \frac{1}{Z_{W_{ik}}} e^{-\beta_{ij} |S_{Fash(i,x,t)} - S_{Fash(k,x,t-1)}|^{B_{ik}}} \quad (39)$$

where in this case, agent $k$ is not influenced by agent $i$. To normalize (13)
It is similar to Advertising in that both are external forces. They act differently, since Advertising pushes the brand up while the weak tie pushes the brand to wherever the weak tie is. The parameter $\beta_{ij}$ must be calculated.

E. Final Model: combining all probabilities

\[
P_{i,x}(\text{Purchase}, \text{Like}(i, x, t), \text{Fash}(i, x, t) | \text{Trust}, \text{Fit}, \text{Price}, \overline{\text{Adv}}) \]

\[
= P_{i,x}(\text{Purchase} | \text{Like}(i, x, t), \text{Fash}(i, x, t), \text{Trust}, \text{Fit}, \text{Price}, \overline{\text{Adv}}) \times P_{i,x}(\text{Like}(i, x, t) | \text{Fash}(i, x, t), \text{Trust}, \text{Fit}, \overline{\text{Adv}})
\]

\[
= P_{i,x}(\text{Purchase} | \text{Like}(i, x, t), \text{Price}) \times P_{i,x}(\text{Like}(i, x, t) | \text{Fash}(i, x, t), \text{Trust}, \text{Fit})
\]

We also consider the approximation

\[
P_{i,x}(\text{Like}(i, x, t), \text{Fash}(i, x, t), \text{Trust}, \text{Fit}, \text{Price}, \overline{\text{Adv}}) = P_{i,x}(\text{Like}(i, x, t) | \text{Fash}(i, x, t), \text{Trust}, \text{Fit})
\]

since (i) Price does not affect how much one likes a pair of jeans $x$, and (ii) Advertising will affect the sense of Fashion and that will affect how much one likes it, but given Fashion, Advertising does not affect further how much one likes a pair of jeans $x$. We further considered from (10)

\[
P_{i,x}(\text{Like}(i, x, t) | \text{Fash}(i, x, t), \text{Trust}(x), \text{Fit}(x))
\]

\[
= \frac{1}{Z_{\text{Like}}} P_{i,x}(\text{Like}(i, x, t) | \text{Fash}(i, x, t), \text{Trust}(x), \text{Fit}(x))
\]

\[
= \frac{1}{Z_{\text{Like}}} P_{\text{Trust}}(\text{Like}(i, x, t) | \text{Trust}(x)) P_{\text{Fit}}(\text{Like}(i, x, t) | \text{Fit}(x)) P_{\text{Fash}}(\text{Like}(i, x, t) | \text{Fash}(i, x, t))
\]

We also consider

\[
P_{i,x}(\text{Fash}(i, x, t) | \text{Fit}, \text{Trust}, \text{Price}, \overline{\text{Adv}}(t)) = P_{i,x}(\text{Fash}(i, x, t) | \overline{\text{Adv}}(t))
\]

since neither of the other variable affect the Fashion variable. Note that Fit and Trust will affect “Like”, i.e., Like will incorporate fashion as well as trust as well as Fit. So our model (37) can be written as

\[
P_{i,x}(\text{Purchase}, \text{Like}(t), \text{Fash}(t) | \text{Trust}, \text{Fit}, \text{Price}, \overline{\text{Adv}}(t))
\]

\[
= P_{i,x}(\text{Purchase} | \text{Price}) \times P_{i,x}(\text{Purchase} | \text{Like}(i, x, t)) \times \frac{1}{Z_{\text{Like}}} P_{\text{Trust}}(\text{Like}(i, x, t) | \text{Trust}(x)) P_{\text{Fit}}(\text{Like}(i, x, t) | \text{Fit}(x)) P_{\text{Fash}}(\text{Like}(i, x, t) | \text{Fash}(i, x, t)) \times P_{i,x}(\text{Fash}(i, x, t) | \overline{\text{Adv}}(x, t))
\]