Homework Assignment #7,

Assigned TUE APR 24, DUE TUE MAY 1, 11:59pm

The sieve of Eratosthenes is a method of generating a list of consecutive prime numbers without having to start a loop beginning with 2 each time. Please see the ppt slides at “Chapter 7, Lists, the Sieve of Eratosthenes.”

Here is how it works:

1. Create a list \( x \) of \( \text{int} \), that has \( N \) elements and let \( N \) be 1000. Set all of the elements of the list to zero, i.e., \( x = 1000*[0] \).

2. In a \texttt{for} loop, starting with the loop index set to 2, test if the list element with that index is set to zero – that means it is a prime number. If it is, in a \texttt{while} loop nested in this \texttt{for} loop, set all the elements of the list to 1 that have subscripts that are multiples of the starting index. These elements represent composite numbers. So when the loop index is 2, you would set \( x[4], x[6], x[8], \) etc., to 1.

3. Each successive value of the \texttt{for} loop should repeat this process until the \texttt{for} loop upper limit \( \sqrt{N} \) is reached.

4. When the loop is complete, all the list elements with value 0 will have a loop index that is a prime number.

5. C. F. Gauss hypothesized that \( \Pi(N) \) the number of primes less than or equal to \( N \) is defined as \( \Pi(N) = N/\log_e(N) \) as \( N \) approaches infinity. This was called the prime number hypothesis.\(^1\) In another \texttt{for} loop print the prime numbers, a counter indicating its ordinal number (1, 2, 3, etc.) and the value of \( \Pi(N) \). The variable \( N \) is the loop index which is the same as the prime number. Here, the maximum value of the loop index should be 1000.

Your output should look something like the following. Don’t worry about labeling the columns exactly as is shown here.

<table>
<thead>
<tr>
<th>Prime number</th>
<th>total num of primes</th>
<th>predicted # of primes</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>11</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

Note that the \( \log_e(N) \) and \( \text{sqrt} \) functions are part of the \texttt{math} module, i.e., \texttt{math.log()} and \texttt{math.sqrt()}; to round the predicted number of primes and the square root, use the built-in function \texttt{round()} \(^2\). If you don’t round \texttt{math.sqrt()} (to an \texttt{int}), you cannot use it in a \texttt{for j in range()} loop.

\(^1\)It was proved independently by Hadamard and de la Vallee Poussin in 1896 and is now called the prime number theorem.